# AMC 10

2002

## P

-1 This test and the matching AMC 12P were developed for the use of a group of Taiwan schools, in early January of 2002. When Taiwan had taken the contests, the AMC released the questions here as a set of practice questions for the 2002 AMC 10 and AMC 12 contests.

1 The ratio $\frac{(2^4)^8}{(4^8)^2}$ equals

**(A)** $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** 8

2 The sum of eleven consecutive integers is 2002. What is the smallest of these integers?

**(A)** 175 **(B)** 177 **(C)** 179 **(D)** 180 **(E)** 181

3 Mary typed a six-digit number, but the two 1s she typed didn't show. What appeared was 2002. How many different six-digit numbers could she have typed?

**(A)** 4 **(B)** 8 **(C)** 10 **(D)** 15 **(E)** 20

4 Which of the following numbers is a perfect square?

**(A)** $4^4 5^5 6^6$ **(B)** $4^4 5^6 6^5$ **(C)** $4^5 5^4 6^6$ **(D)** $4^6 5^4 6^5$ **(E)** $4^6 5^5 6^4$

5 Let $(a_n)_{n\geq 1}$ be a sequence such that $a_1 = 1$ and $3a_{n+1} - 3a_n = 1$ for all $n \geq 1$. Find $a_{2002}$.

**(A)** 666 **(B)** 667 **(C)** 668 **(D)** 669 **(E)** 670

6 The perimeter of a rectangle is 100 and its diagonal has length $x$. What is the area of this rectangle?

**(A)** $625 - x^2$ **(B)** $625 - \frac{x^2}{2}$ **(C)** $1250 - x^2$ **(D)** $1250 - \frac{x^2}{2}$ **(E)** $2500 - \frac{x^2}{2}$

7 The dimensions of a rectangular box in inches are all positive integers and the volume of the box is 2002 $\text{in}^3$. Find the minimum possible sum in inches of the three dimensions.

**(A)** 36 **(B)** 38 **(C)** 42 **(D)** 44 **(E)** 92

8 How many ordered triples of positive integers $(x, y, z)$ satisfy $(x^y)^z = 64$?

**(A)** 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

9 The function $f$ is given by the table

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | 4 | 1 | 3 | 5 | 2 |

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If $u_0 = 4$ and $u_{n+1} = f(u_n)$ for $n \geq 0$, find $u_{2002}$.

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

10 Let $a$ and $b$ be distinct real numbers for which

$$\frac{a}{b} + \frac{a+10b}{b+10a} = 2.$$

Find $\frac{a}{b}$.

**(A)** 0.6 **(B)** 0.7 **(C)** 0.8 **(D)** 0.9 **(E)** 1

11 Let $P(x) = kx^3 + 2k^2x^2 + k^3$. Find the sum of all real numbers $k$ for which $x - 2$ is a factor of $P(x)$.

**(A)** $-8$ **(B)** $-4$ **(C)** 0 **(D)** 5 **(E)** 8

12 For $f_n(x) = x^n$ and $a \neq 1$ consider

I. $(f_{11}(a)f_{13}(a))^{14}$

II. $f_{11}(a)f_{13}(a)f_{14}(a)$

III. $(f_{11}(f_{13}(a)))^{14}$

IV. $f_{11}(f_{13}(f_{14}(a)))$

Which of these equal $f_{2002}(a)$?

**(A)** I and II only **(B)** II and III only **(C)** III and IV only **(D)** II, III, and IV only **(E)** all of them

13 Participation in the local soccer league this year is 10% higher than last year. The number of males increased by 5% and the number of females increased by 20%. What fraction of the soccer league is now female?

**(A)** $\frac{1}{3}$ **(B)** $\frac{4}{11}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{4}{9}$ **(E)** $\frac{1}{2}$

14 The vertex $E$ of a square $EFGH$ is at the center of square $ABCD$. The length of a side of $ABCD$ is 1 and the length of a side of $EFGH$ is 2. Side $EF$ intersects $CD$ at $I$ and $EH$ intersects $AD$ at $J$. If angle $EID = 60°$, the area of quadrilateral $EIDJ$ is

**(A)** $\frac{1}{4}$ **(B)** $\frac{\sqrt{3}}{6}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{\sqrt{2}}{4}$ **(E)** $\frac{\sqrt{3}}{2}$

15 What is the smallest integer $n$ for which any subset of $\{1, 2, 3, \ldots, 20\}$ of size $n$ must contain two numbers that differ by 8?

**(A)** 2 **(B)** 8 **(C)** 12 **(D)** 13 **(E)** 15

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16 Two walls and the ceiling of a room meet at right angles at point $P$. A fly is in the air one meter from one wall, eight meters from the other wall, and 9 meters from point $P$. How many meters is the fly from the ceiling?

**(A)** $\sqrt{13}$ **(B)** $\sqrt{14}$ **(C)** $\sqrt{15}$ **(D)** 4 **(E)** $\sqrt{17}$

17 There are 1001 red marbles and 1001 black marbles in a box. Let $P_s$ be the probability that two marbles drawn at random from the box are the same color, and let $P_d$ be the probability that they are different colors. Find $|P_s - P_d|$.

**(A)** 0 **(B)** $\frac{1}{2002}$ **(C)** $\frac{1}{2001}$ **(D)** $\frac{2}{2001}$ **(E)** $\frac{1}{1000}$

18 For how many positive integers $n$ is $n^3 - 8n^2 + 20n - 13$ a prime number?

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** more than 4

19 If $a, b, c$ are real numbers such that $a^2 + 2b = 7$, $b^2 + 4c = -7$, and $c^2 + 6a = -14$, find $a^2 + b^2 + c^2$.

**(A)** 14 **(B)** 21 **(C)** 28 **(D)** 35 **(E)** 49

20 How many three-digit numbers have at least one 2 and at least one 3?

**(A)** 52 **(B)** 54 **(C)** 56 **(D)** 58 **(E)** 60

21 Let $f$ be a real-valued function such that

$$f(x) + 2f\left(\frac{2002}{x}\right) = 3x$$

for all $x > 0$. Find $f(2)$.

**(A)** 1000 **(B)** 2000 **(C)** 3000 **(D)** 4000 **(E)** 6000

22 In how many zeroes does the number $\frac{2002!}{(1001!)^2}$ end?

**(A)** 0 **(B)** 1 **(C)** 2 **(D)** 200 **(E)** 400

23 Let

$$a = \frac{1^2}{1} + \frac{2^2}{3} + \frac{3^2}{5} + \cdots + \frac{1001^2}{2001}$$

and

$$b = \frac{1^2}{3} + \frac{2^2}{5} + \frac{3^2}{7} + \cdots + \frac{1001^2}{2003}.$$

Find the integer closest to $a - b$.

**(A)** 500 **(B)** 501 **(C)** 999 **(D)** 1000 **(E)** 1001

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24 What is the maximum value of $n$ for which there is a set of distinct positive integers $k_1, k_2, \ldots, k_n$ for which

$$k_1^2 + k_2^2 + \ldots + k_n^2 = 2002?$$

**(A)** 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

25 Under the new AMC 10, 12 scoring method, 6 points are given for each correct answer, 2.5 points are given for each unanswered question, and no points are given for an incorrect answer. Some of the possible scores between 0 and 150 can be obtained in only one way, for example, the only way to obtain a score of 146.5 is to have 24 correct answers and one unanswered question. Some scores can be obtained in exactly two ways; for example, a score of 104.5 can be obtained with 17 correct answers, 1 unanswered question, and 7 incorrect, and also with 12 correct answers and 13 unanswered questions. There are three scores that can be obtained in exactly three ways. What is their sum?

**(A)** 175 **(B)** 179.5 **(C)** 182 **(D)** 188.5 **(E)** 201

# 2002 AMC 10P Answer Key

1. (C)
2. (B)
3. (D)
4. (C)
5. (C)
6. (D)
7. (B)
8. (E)
9. (B)
10. (C)
11. (A)
12. (C)
13. (B)
14. (A)
15. (D)
16. (D)
17. (C)
18. (C)
19. (A)
20. (A)
21. (B)
22. (B)
23. (B)
24. (D)
25. (D)