AMC 12/AHSME

2005

B

1 A scout troop buys 1000 candy bars at a price of ve for \$2. They sell all the candy bars at a price of two for \$1. What was their prot, in dollars?

(A) 100 **(B)** 200 **(C)** 300 **(D)** 400 **(E)** 500

2 A positive number x has the property that $x\%$ of x is 4. What is x?

(A) 2 **(B)** 4 **(C)** 10 **(D)** 20 **(E)** 40

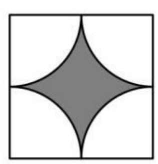
3 Brianna is using part of the money she earned on her weekend job to buy several equally-priced CDs. She used one fth of her money to buy one third of the CDs. What fraction of her money will she have left after she buys all the CDs?

(A) $\frac{1}{5}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{4}{5}$

4 At the beginning of the school year, Lisas goal was to earn an A on at least 80% of her 50 quizzes for the year. She earned an A on 22 of the rst 30 quizzes. If she is to achieve her goal, on at most how many of the remaining quizzes can she earn a grade lower than an A?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

5 An 8-foot by 10-foot oor is tiled with square tiles of size 1 foot by 1 foot. Each tile has a pattern consisting of four white quarter circles of radius 1/2 foot centered at each corner of the tile. The remaining portion of the tile is shaded. How many square feet of the oor are shaded?



(A) $80-20\pi$ **(B)** $60-10\pi$ **(C)** $80-10\pi$ **(D)** $60+10\pi$ **(E)** $80+10\pi$

6 In $\triangle ABC$, we have $AC = BC = 7$ and $AB = 2$. Suppose that D is a point on line AB such that B lies between A and D and $CD = 8$. What is BD?

(A) 3 **(B)** $2\sqrt{3}$ **(C)** 4 **(D)** 5 **(E)** $4\sqrt{2}$

7 What is the area enclosed by the graph of $|3x| + |4y| = 12$?

(A) 6 **(B)** 12 **(C)** 16 **(D)** 24 **(E)** 25

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8 For how many values of a is it true that the line $y = x + a$ passes through the vertex of the parabola $y = x^2 + a^2$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 10 **(E)** infinitely many

9 On a certain math exam, 10% of the students got 70 points, 25% got 80 points, 20% got 85 points, 15% got 90 points, and the rest got 95 points. What is the difference between the mean and the median score on this exam?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 5

10 The first term of a sequence is 2005. Each succeeding term is the sum of the cubes of the digits of the previous terms. What is the 2005th term of the sequence?

(A) 29 **(B)** 55 **(C)** 85 **(D)** 133 **(E)** 250

11 An envelope contains eight bills: 2 ones, 2 fives, 2 tens, and 2 twenties. Two bills are drawn at random without replacement. What is the probability that their sum is \$20 or more?

(A) $\frac{1}{4}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{3}{7}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

12 The quadratic equation $x^2 + mx + n = 0$ has roots that are twice those of $x^2 + px + m = 0$, and none of m, n, and p is zero. What is the value of n/p?

(A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

13 Suppose that $4^{x_1} = 5, 5^{x_2} = 6, 6^{x_3} = 7, ..., 127^{x_{124}} = 128$. What is $x_1 x_2 \cdots x_{124}$?

(A) 2 **(B)** $\frac{5}{2}$ **(C)** 3 **(D)** $\frac{7}{2}$ **(E)** 4

14 A circle having center $(0, k)$, with $k > 6$, is tangent to the lines $y = x, y = -x$ and $y = 6$. What is the radius of this circle?

(A) $6\sqrt{2} - 6$ **(B)** 6 **(C)** $6\sqrt{2}$ **(D)** 12 **(E)** $6 + 6\sqrt{2}$

15 The sum of four two-digit numbers is 221. None of the eight digits is 0 and no two of them are same. Which of the following is **not** included among the eight digits?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

16 Eight spheres of radius 1, one per octant, are each tangent to the coordinate planes. What is the radius of the smallest sphere, centered at the origin, thta contains these eight spheres?

(A) $\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** $1 + \sqrt{2}$ **(D)** $1 + \sqrt{3}$ **(E)** 3

17 How many distinct four-tuples (a, b, c, d) of rational numbers are there with

$a \log_{10} 2 + b \log_{10} 3 + c \log_{10} 5 + d \log_{10} 7 = 2005$?

(A) 0 **(B)** 1 **(C)** 17 **(D)** 2004 **(E)** infinitely many

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18 Let $A(2,2)$ and $B(7,7)$ be points in the plane. Define R as the region in the first quadrant consisting of those points C such that $\triangle ABC$ is an acute triangle. What is the closest integer to the area of the region R?

(A) 25 **(B)** 39 **(C)** 51 **(D)** 60 **(E)** 80

19 Let x and y be two-digit integers such that y is obtained by reversing the digits of x. The integers x and y satisfy $x^2 - y^2 = m^2$ for some positive integer m. What is $x + y + m$?

(A) 88 **(B)** 112 **(C)** 116 **(D)** 144 **(E)** 154

20 Let a, b, c, d, e, f, g and h be distinct elements in the set

$$\{-7, -5, -3, -2, 2, 4, 6, 13\}.$$

What is the minimum possible value of

$$(a + b + c + d)^2 + (e + f + g + h)^2$$

(A) 30 **(B)** 32 **(C)** 34 **(D)** 40 **(E)** 50

21 A positive integer n has 60 divisors and $7n$ has 80 divisors. What is the greatest integer k such that 7^k divides n?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

22 A sequence of complex numbers $z_0, z_1, z_2, ...$ is defined by the rule

$$z_{n+1} = \frac{iz_n}{\overline{z_n}}$$

where $\overline{z_n}$ is the complex conjugate of z_n and $i^2 = -1$. Suppose that $|z_0| = 1$ and $z_{2005} = 1$. How many possible values are there for z_0?

(A) 1 **(B)** 2 **(C)** 4 **(D)** 2005 **(E)** 2^{2005}

23 Let S be the set of ordered triples (x, y, z) of real numbers for which

$$\log_{10}(x + y) = z \text{ and } \log_{10}(x^2 + y^2) = z + 1.$$

There are real numbers a and b such that for all ordered triples (x, y, z) in S we have $x^3 + y^3 = a \cdot 10^{3z} + b \cdot 10^{2z}$. What is the value of $a + b$?

(A) $\frac{15}{2}$ **(B)** $\frac{29}{2}$ **(C)** 15 **(D)** $\frac{39}{2}$ **(E)** 24

24 All three vertices of an equilateral triangle are on the parabola $y = x^2$, and one of its sides has a slope of 2. The x-coordinates of the three vertices have a sum of m/n, where m and n are relatively prime positive integers. What is the value of $m + n$?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

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25 Six ants simultaneously stand on the six vertices of a regular octahedron, with each ant at a different vertex. Simultaneously and independently, each ant moves from its vertex to one of the four adjacent vertices, each with equal probability. What is the probability that no two ants arrive at the same vertex?

(A) $\frac{5}{256}$ **(B)** $\frac{21}{1024}$ **(C)** $\frac{11}{512}$ **(D)** $\frac{23}{1024}$ **(E)** $\frac{3}{128}$

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2005 AMC 12B Answer Key

1. A
2. D
3. C
4. B
5. A
6. A
7. D
8. C
9. B
10. E
11. D
12. D
13. D
14. E
15. D
16. D
17. B
18. C
19. E
20. C
21. C
22. E
23. B
24. A
25. A