AMC 12/AHSME

2009

A

1 Kim's flight took off from Newark at 10:34 AM and landed in Miami at 1:18 PM. Both cities are in the same time zone. If her flight took h hours and m minutes, with $0 < m < 60$, what is $h + m$?

(A) 46 **(B)** 47 **(C)** 50 **(D)** 53 **(E)** 54

2 Which of the following is equal to $1 + \frac{1}{1+\frac{1}{1+1}}$?

(A) $\frac{5}{4}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{5}{3}$ **(D)** 2 **(E)** 3

3 What number is one third of the way from $\frac{1}{4}$ to $\frac{3}{4}$?

(A) $\frac{1}{3}$ **(B)** $\frac{5}{12}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{7}{12}$ **(E)** $\frac{2}{3}$

4 Four coins are picked out of a piggy bank that contains a collection of pennies, nickels, dimes, and quarters. Which of the following could *not* be the total value of the four coins, in cents?

(A) 15 **(B)** 25 **(C)** 35 **(D)** 45 **(E)** 55

5 One dimension of a cube is increased by 1, another is decreased by 1, and the third is left unchanged. The volume of the new rectangular solid is 5 less than that of the cube. What was the volume of the cube?

(A) 8 **(B)** 27 **(C)** 64 **(D)** 125 **(E)** 216

6 Suppose that $P = 2^m$ and $Q = 3^n$. Which of the following is equal to 12^{mn} for every pair of integers (m, n)?

(A) P^2Q **(B)** P^nQ^m **(C)** P^nQ^{2m} **(D)** $P^{2m}Q^n$ **(E)** $P^{2n}Q^m$

7 The first three terms of an arithmetic sequence are $2x - 3$, $5x - 11$, and $3x + 1$ respectively. The nth term of the sequence is 2009. What is n?

(A) 255 **(B)** 502 **(C)** 1004 **(D)** 1506 **(E)** 8037

8 Four congruent rectangles are placed as shown. The area of the outer square is 4 times that of the inner square. What is the ratio of the length of the longer side of each rectangle to the length of its shorter side?

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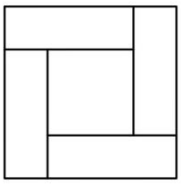
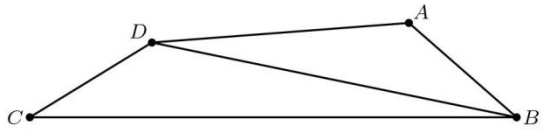


(A) 3 **(B)** $\sqrt{10}$ **(C)** $2+\sqrt{2}$ **(D)** $2\sqrt{3}$ **(E)** 4

9 Suppose that $f(x+3) = 3x^2 + 7x + 4$ and $f(x) = ax^2 + bx + c$. What is $a + b + c$?

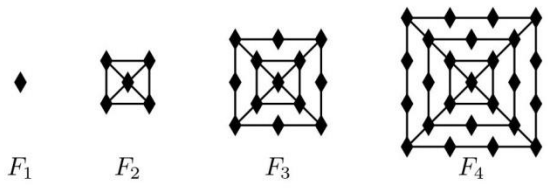
(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** 3

10 In quadrilateral $ABCD$, $AB = 5$, $BC = 17$, $CD = 5$, $DA = 9$, and BD is an integer. What is BD?



(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

11 The figures F_1, F_2, F_3, and F_4 shown are the first in a sequence of figures. For $n \geq 3$, F_n is constructed from F_{n-1} by surrounding it with a square and placing one more diamond on each side of the new square than F_{n-1} had on each side of its outside square. For example, figure F_3 has 13 diamonds. How many diamonds are there in figure F_{20}?



(A) 401 **(B)** 485 **(C)** 585 **(D)** 626 **(E)** 761

12 How many positive integers less than 1000 are 6 times the sum of their digits?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 12

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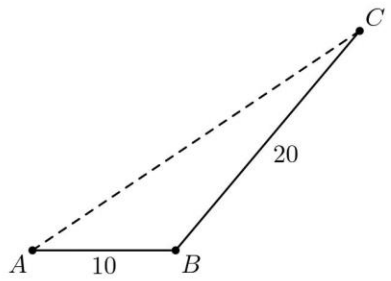
13 A ship sails 10 miles in a straight line from A to B, turns through an angle between $45°$ and $60°$, and then sails another 20 miles to C. Let AC be measured in miles. Which of the following intervals contains AC^2?



(A) $[400, 500]$ **(B)** $[500, 600]$ **(C)** $[600, 700]$ **(D)** $[700, 800]$ **(E)** $[800, 900]$

14 A triangle has vertices $(0, 0)$, $(1, 1)$, and $(6m, 0)$, and the line $y = mx$ divides the triangle into two triangles of equal area. What is the sum of all possible values of m?

(A) $-\frac{1}{3}$ **(B)** $-\frac{1}{6}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

15 For what value of n is $i + 2i^2 + 3i^3 + \cdots + ni^n = 48 + 49i$?

Note: here $i = \sqrt{-1}$.

(A) 24 **(B)** 48 **(C)** 49 **(D)** 97 **(E)** 98

16 A circle with center C is tangent to the positive x and y-axes and externally tangent to the circle centered at $(3, 0)$ with radius 1. What is the sum of all possible radii of the circle with center C?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 9

17 Let $a + ar_1 + ar_1^2 + ar_1^3 + \cdots$ and $a + ar_2 + ar_2^2 + ar_2^3 + \cdots$ be two different infinite geometric series of positive numbers with the same first term. The sum of the first series is r_1, and the sum of the second series is r_2. What is $r_1 + r_2$?

(A) 0 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** $\frac{1+\sqrt{5}}{2}$ **(E)** 2

18 For $k > 0$, let $I_k = 10\ldots064$, where there are k zeros between the 1 and the 6. Let $N(k)$ be the number of factors of 2 in the prime factorization of I_k. What is the maximum value of $N(k)$?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

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19 Andrea inscribed a circle inside a regular pentagon, circumscribed a circle around the pentagon, and calculated the area of the region between the two circles. Bethany did the same with a regular heptagon (7 sides). The areas of the two regions were A and B, respectively. Each polygon had a side length of 2. Which of the following is true?

(A) $A = \frac{25}{49}B$ **(B)** $A = \frac{5}{7}B$ **(C)** $A = B$ **(D)** $A = \frac{7}{5}B$ **(E)** $A = \frac{49}{25}B$

20 Convex quadrilateral $ABCD$ has $AB = 9$ and $CD = 12$. Diagonals AC and BD intersect at E, $AC = 14$, and $\triangle AED$ and $\triangle BEC$ have equal areas. What is AE?

(A) $\frac{9}{2}$ **(B)** $\frac{50}{11}$ **(C)** $\frac{21}{4}$ **(D)** $\frac{17}{3}$ **(E)** 6

21 Let $p(x) = x^3 + ax^2 + bx + c$, where a, b, and c are complex numbers. Suppose that

$$p(2009 + 9002\pi i) = p(2009) = p(9002) = 0$$

What is the number of nonreal zeros of $x^{12} + ax^8 + bx^4 + c$?

(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

22 A regular octahedron has side length 1. A plane parallel to two of its opposite faces cuts the octahedron into the two congruent solids. The polygon formed by the intersection of the plane and the octahedron has area $\frac{a\sqrt{b}}{c}$, where a, b, and c are positive integers, a and c are relatively prime, and b is not divisible by the square of any prime. What is $a + b + c$?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

23 Functions f and g are quadratic, $g(x) = -f(100 - x)$, and the graph of g contains the vertex of the graph of f. The four x-intercepts on the two graphs have x-coordinates x_1, x_2, x_3, and x_4, in increasing order, and $x_3 - x_2 = 150$. The value of $x_4 - x_1$ is $m + n\sqrt{p}$, where m, n, and p are positive integers, and p is not divisible by the square of any prime. What is $m + n + p$?

(A) 602 **(B)** 652 **(C)** 702 **(D)** 752 **(E)** 802

24 The *tower function of twos* is defined recursively as follows: $T(1) = 2$ and $T(n + 1) = 2^{T(n)}$ for $n \geq 1$. Let $A = (T(2009))^{T(2009)}$ and $B = (T(2009))^A$. What is the largest integer k such that

$$\underbrace{\log_2 \log_2 \log_2 \ldots \log_2 B}_{k \text{ times}}$$

is defined?

(A) 2009 **(B)** 2010 **(C)** 2011 **(D)** 2012 **(E)** 2013

25 The first two terms of a sequence are $a_1 = 1$ and $a_2 = \frac{1}{\sqrt{3}}$. For $n \geq 1$,

$$a_{n+2} = \frac{a_n + a_{n+1}}{1 - a_n a_{n+1}}.$$

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What is $|a_{2009}|$?

(A) 0 **(B)** $2-\sqrt{3}$ **(C)** $\frac{1}{\sqrt{3}}$ **(D)** 1 **(E)** $2+\sqrt{3}$

2009 AMC 12A Answer Key

1. A
2. C
3. B
4. A
5. D
6. E
7. B
8. A
9. D
10. C
11. E
12. B
13. D
14. B
15. D
16. D
17. C
18. B
19. C
20. E
21. C
22. E
23. D
24. E
25. A