AMC 12/AHSME
2012

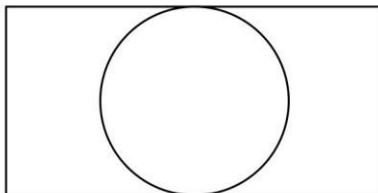## B

1. Each third-grade classroom at Pearl Creek Elementary has 18 students and 2 pet rabbits. How many more students than rabbits are there in all 4 of the third-grade classrooms?

   (A) 48   **(B)** 56   **(C)** 64   **(D)** 72   **(E)** 80

2. A circle of radius 5 is inscribed in a rectangle as shown. The ratio of the the length of the rectangle to its width is 2 : 1. What is the area of the rectangle?

   (A) 50   **(B)** 100   **(C)** 125   **(D)** 150   **(E)** 200

3. For a science project, Sammy observed a chipmunk and a squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide?

   (A) 30   **(B)** 36   **(C)** 42   **(D)** 48   **(E)** 54

4. Suppose that the euro is worth 1.30 dollars. If Diana has 500 dollars and Etienne has 400 euros, by what percent is the value of Etienne's money greater than the value of Diana's money?

   (A) 2   **(B)** 4   **(C)** 6.5   **(D)** 8   **(E)** 13

5. Two integers have a sum of 26. When two more integers are added to the first two integers the sum is 41. Finally when two more integers are added to the sum of the previous four integers the sum is 57. What is the minimum number of even integers among the 6 integers?

   (A) 1   **(B)** 2   **(C)** 3   **(D)** 4   **(E)** 5

6. In order to estimate the value of $x - y$ where x and y are real numbers with $x > y > 0$, Xiaoli rounded x up by a small amount, rounded y down by the same amount, and then subtracted her values. Which of the following statements is necessarily correct?

(A) Her estimate is larger than $x - y$
(B) Her estimate is smaller than $x - y$
(C) Her estimate equals $x - y$
(D) Her estimate equals $y - x$
(E) Her estimate is 0

7. Small lights are hung on a string 6 inches apart in the order red, red, green, green, green, red, red, green, green, green, and so on continuing this pattern of 2 red lights followed by 3 green lights. How many feet separate the third red light and the 21st red light?

Note: 1 foot is equal to 12 inches.

(A) 18   **(B)** 18.5   **(C)** 20   **(D)** 20.5   **(E)** 22.5

8. A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

(A) 729   **(B)** 972   **(C)** 1024   **(D)** 2187   **(E)** 2304

9. It takes Clea 60 seconds to walk down an escalator when it is not operating and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

(A) 36   **(B)** 40   **(C)** 42   **(D)** 48   **(E)** 52

10. What is the area of the polygon whose vertices are the points of intersection of the curves $x^2 + y^2 = 25$ and $(x - 4)^2 + 9y^2 = 81$?

(A) 24   **(B)** 27   **(C)** 36   **(D)** 37.5   **(E)** 42

11. In the equation below, A and B are consecutive positive integers, and A, B, and $A + B$ represent number bases:

$$132_A + 43_B = 69_{A+B}.$$

What is $A + B$?

(A) 9   **(B)** 11   **(C)** 13   **(D)** 15   **(E)** 17

12. How many sequences of zeros and ones of length 20 have all the zeros consecutive, or all the ones consecutive, or both?

(A) 190   **(B)** 192   **(C)** 211   **(D)** 380   **(E)** 382

13. Two parabolas have equations $y = x^2 + ax + b$ and $y = x^2 + cx + d$, where a, b, c, and d are integers (not necessarily different), each chosen independently by rolling a fair six-sided die. What is the probability that the parabolas have at least one point in common?

(A) $\frac{1}{2}$ (B) $\frac{25}{36}$ (C) $\frac{5}{6}$ (D) $\frac{31}{36}$ (E) 1

14. Bernado and Silvia play the following game. An integer between 0 and 999, inclusive, is selected and given to Bernado. Whenever Bernado receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she adds 50 to it and passes the result to Bernado. The winner is the last person who produces a number less than 1000. Let N be the smallest initial number that results in a win for Bernado. What is the sum of the digits of N?

(A) 7   (B) 8   (C) 9   (D) 10   (E) 11

15. Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger?

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{\sqrt{10}}{10}$ (D) $\frac{\sqrt{5}}{6}$ (E) $\frac{\sqrt{10}}{5}$

16. Amy, Beth, and Jo listen to four different songs and discuss which ones they like. No song is liked by all three. Furthermore, for each of the three pairs of the girls, there is at least one song liked by those two girls but disliked by the third. In how many different ways is this possible?

(A) 108   (B) 132   (C) 671   (D) 846   (E) 1105

17. Square $PQRS$ lies in the first quadrant. Points $(3,0),(5,0),(7,0)$, and $(13,0)$ lie on lines SP, RQ, PQ, and SR, respectively. What is the sum of the coordinates of the center of the square $PQRS$?

(A) 6   (B) 6.2   (C) 6.4   (D) 6.6   (E) 6.8

18. Let $(a_1, a_2, \ldots, a_{10})$ be a list of the first 10 positive integers such that for each $2 \le i \le 10$ either $a_i + 1$ or $a_i - 1$ or both appear somewhere before a_i in the list. How many such lists are there?

(A) 120   (B) 512   (C) 1024   (D) 181,440   (E) 362,880

19. A unit cube has vertices $P_1, P_2, P_3, P_4, P_1', P_2', P_3'$, and P_4'. Vertices P_2, P_3, and P_4 are adjacent to P_1, and for $1 \le i \le 4$, vertices P_i and P_i' are opposite to each other. A regular octahedron has one vertex in each of the segments $P_1P_2, P_1P_3, P_1P_4, P_1'P_2', P_1'P_3'$, and $P_1'P_4'$. What is the octahedron's side length?

(A) $\frac{3\sqrt{2}}{4}$ (B) $\frac{7\sqrt{6}}{16}$ (C) $\frac{\sqrt{5}}{2}$ (D) $\frac{2\sqrt{3}}{3}$ (E) $\frac{\sqrt{6}}{2}$

20. A trapezoid has side lengths $3, 5, 7$, and 11. The sum of all the possible areas of the trapezoid can be written in the form of $r_1\sqrt{n_1} + r_2\sqrt{n_2} + r_3$, where r_1, r_2, and r_3 are rational numbers

and n_1 and n_2 are positive integers not divisible by the square of a prime. What is the greatest integer less than or equal to
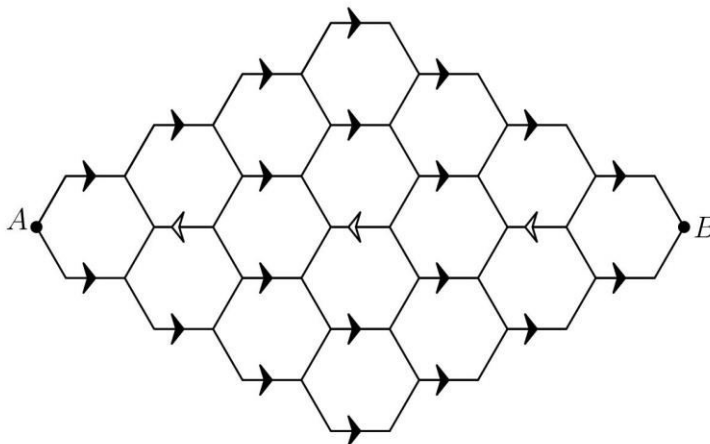
$$r_1 + r_2 + r_3 + n_1 + n_2?$$

(A) 57 **(B)** 59 **(C)** 61 **(D)** 63 **(E)** 65

21 Square $AXYZ$ is inscribed in equiangular hexagon $ABCDEF$ with X on $\overline{BC}$, Y on $\overline{DE}$, and Z on $\overline{EF}$. Suppose that $AB = 40$, and $EF = 41(\sqrt{3}-1)$. What is the side-length of the square?

(A) $29\sqrt{3}$ **(B)** $\frac{21}{2}\sqrt{2} + \frac{41}{2}\sqrt{3}$ **(C)** $20\sqrt{3} + 16$ **(D)** $20\sqrt{2} + 13\sqrt{3}$ **(E)** $21\sqrt{6}$

22 A bug travels from A to B along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once. How many different paths are there?

(A) 2112 **(B)** 2304 **(C)** 2368 **(D)** 2384 **(E)** 2400

23 Consider all polynomials of a complex variable, $P(z) = 4z^4 + az^3 + bz^2 + cz + d$, where a, b, c and d are integers, $0 \le d \le c \le b \le a \le 4$, and the polynomial has a zero z_0 with $|z_0| = 1$. What is the sum of all values $P(1)$ over all the polynomials with these properties?

(A) 84 **(B)** 92 **(C)** 100 **(D)** 108 **(E)** 120

24 Define the function f_1 on the positive integers by setting $f_1(1) = 1$ and if $n = p_1^{e_1} p_2^{e_2} \ldots p_k^{e_k}$ is the prime factorization of $n > 1$, then:

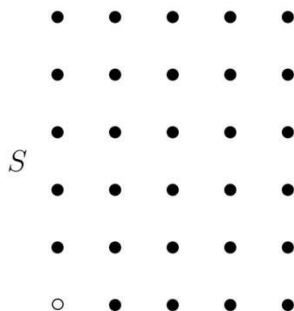$f_1(n) = (p_1 + 1)^{e_1 - 1}(p_2 + 1)^{e_2 - 1}....(p_k + 1)^{e_k - 1}$

For every $m \geq 2$, let $f_m(n) = f_1(f_{m-1}(n))$. For how many N in the range $1 \leq N \leq 400$ is the sequence $(f_1(N), f_2(N), f_3(N), ...)$ unbounded?

Note: a sequence of positive numbers is unbounded if for every integer B, there is a member of the sequence greater than B.

(A) 15   (B) 16   (C) 17   (D) 18   (E) 19

25. Let $S = \{(x, y) : x \in \{0, 1, 2, 3, 4\}, y \in \{0, 1, 2, 3, 4, 5\}, \text{ and } (x, y) \neq (0, 0)\}$. Let T be the set of all right triangles whose vertices are in S. For every right triangle $t = \triangle ABC$ with vertices A, B, and C in counter-clockwise order and right angle at A, let $f(t) = \tan(\angle CBA)$. What is

$$\prod_{t \in T} f(t)?$$

(A) 1   (B) $\frac{625}{144}$ (C) $\frac{125}{24}$ (D) 6   (E) $\frac{625}{24}$

2012 AMC 12A Answer Key

1. E
2. D
3. D
4. C
5. D
6. D
7. C
8. C
9. A
10. D
11. B
12. D
13. D
14. E
15. A
16. E
17. B
18. A
19. B
20. B
21. E
22. C
23. C
24. C
25. C