2019 AMC 10A

Problem 1

What is the value of $2^{(0^{(1^9)})} + ((2^0)^1)^9$?

$\textbf{(A) } 0 \qquad \textbf{(B) } 1 \qquad \textbf{(C) } 2 \qquad \textbf{(D) } 3 \qquad \textbf{(E) } 4$

Problem 2

What is the hundreds digit of $(20! - 15!)$?

$\textbf{(A) } 0 \qquad \textbf{(B) } 1 \qquad \textbf{(C) } 2 \qquad \textbf{(D) } 4 \qquad \textbf{(E) } 5$

Problem 3

Ana and Bonita were born on the same date in different years, n years apart. Last year Ana was 5 times as old as Bonita. This year Ana's age is the square of Bonita's age. What is n?

$\textbf{(A) } 3 \qquad \textbf{(B) } 5 \qquad \textbf{(C) } 9 \qquad \textbf{(D) } 12 \qquad \textbf{(E) } 15$

Problem 4

A box contains 28 red balls, 20 green balls, 19 yellow balls, 13 blue balls, 11 white balls, and 9 black balls. What is the minimum number of balls that must be drawn from the box without replacement to guarantee that at least 15 balls of a single color will be drawn?

$\textbf{(A) } 75 \qquad \textbf{(B) } 76 \qquad \textbf{(C) } 79 \qquad \textbf{(D) } 84 \qquad \textbf{(E) } 91$

Problem 5

What is the greatest number of consecutive integers whose sum is $45?$

$\textbf{(A) } 9 \qquad \textbf{(B) } 25 \qquad \textbf{(C) } 45 \qquad \textbf{(D) } 90 \qquad \textbf{(E) } 120$

Problem 6

For how many of the following types of quadrilaterals does there exist a point in the plane of the quadrilateral that is equidistant from all four vertices of the quadrilateral?

- a square
- a rectangle that is not a square
- a rhombus that is not a square
- a parallelogram that is not a rectangle or a rhombus
- an isosceles trapezoid that is not a parallelogram

$\textbf{(A) } 1 \qquad \textbf{(B) } 2 \qquad \textbf{(C) } 3 \qquad \textbf{(D) } 4 \qquad \textbf{(E) } 5$

Problem 7

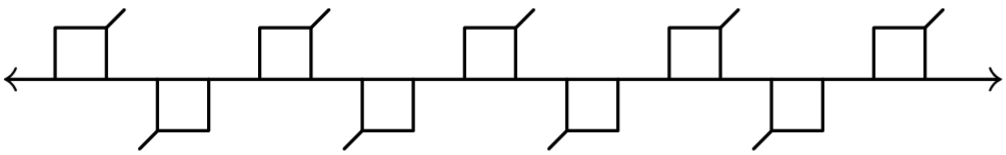
Two lines with slopes $\dfrac{1}{2}$ and 2 intersect at $(2,2)$. What is the area of the triangle enclosed by these two lines and the line

$$x+y=10?$$

$\textbf{(A) } 4 \qquad \textbf{(B) } 4\sqrt{2} \qquad \textbf{(C) } 6 \qquad \textbf{(D) } 8 \qquad \textbf{(E) } 6\sqrt{2}$

Problem 8

The figure below shows line ℓ with a regular, infinite, recurring pattern of squares and line segments.



How many of the following four kinds of rigid motion transformations of the plane in which this figure is drawn, other than the identity transformation, will transform this figure into itself?

- some rotation around a point of line ℓ
- some translation in the direction parallel to line ℓ
- the reflection across line ℓ
- some reflection across a line perpendicular to line ℓ

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Problem 9

What is the greatest three-digit positive integer n for which the sum of the first n positive integers is $\underline{not}$ a divisor of the product of the first n positive integers?

(A) 995 **(B)** 996 **(C)** 997 **(D)** 998 **(E)** 999

Problem 10

A rectangular floor that is 10 feet wide and 17 feet long is tiled with 170 one-foot square tiles. A bug walks from one corner to the opposite corner in a straight line. Including the first and the last tile, how many tiles does the bug visit?

$\textbf{(A)}\ 17 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 26 \qquad \textbf{(D)}\ 27 \qquad \textbf{(E)}\ 28$

Problem 11

How many positive integer divisors of 201^9 are perfect squares or perfect cubes (or both)?

$\textbf{(A)}\ 32 \qquad \textbf{(B)}\ 36 \qquad \textbf{(C)}\ 37 \qquad \textbf{(D)}\ 39 \qquad \textbf{(E)}\ 41$

Problem 12

Melanie computes the mean μ, the median M, and the modes of the 365 values that are the dates in the months of 2019. Thus her data consist of twelve 1s, twelve 2s, . . . , twelve 28s, eleven 29s, eleven 30s, and seven 31s. Let d be the median of the modes. Which of the following statements is true?

$\textbf{(A)}\ \mu < d < M \qquad \textbf{(B)}\ M < d < \mu \qquad \textbf{(C)}\ d = M = \mu \qquad \textbf{(D)}\ d < M < \mu \qquad \textbf{(E)}\ d < \mu < M$

Problem 13

How many of the first 2018 numbers in the sequence $101, 1001, 10001, 100001, \dots$ are divisible by 101?

$\textbf{(A)}\ 253 \qquad \textbf{(B)}\ 504 \qquad \textbf{(C)}\ 505 \qquad \textbf{(D)}\ 506 \qquad \textbf{(E)}\ 1009$

Problem 14

For a set of four distinct lines in a plane, there are exactly N distinct points that lie on two or more of the lines. What is the sum of all possible values of N?

$\textbf{(A)}\ 14 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 19 \qquad \textbf{(E)}\ 21$

Problem 15

A sequence of numbers is defined recursively by $a_1 = 1$, $a_2 = \frac{3}{7}$, and

$$a_n = \frac{a_{n-2} \cdot a_{n-1}}{2a_{n-2} - a_{n-1}}$$

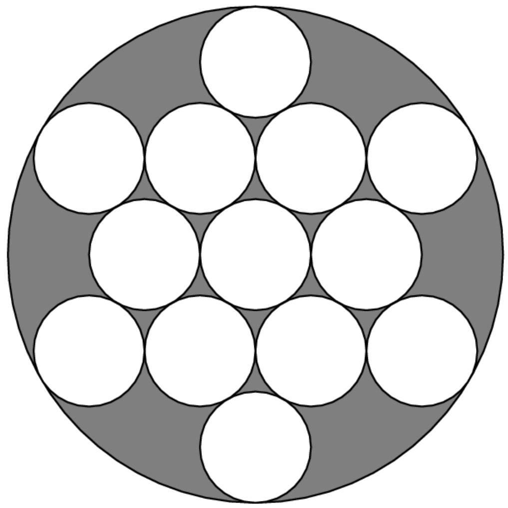
for all $n \geq 3$.

Then a_{2019} can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p+q$?

$\textbf{(A)}\ 2020 \qquad \textbf{(B)}\ 4039 \qquad \textbf{(C)}\ 6057 \qquad \textbf{(D)}\ 6061 \qquad \textbf{(E)}\ 8078$

Problem 16

The figure below shows 13 circles of radius 1 within a larger circle. All the intersections occur at points of tangency. What is the area of the region, shaded in the figure, inside the larger circle but outside all the circles of radius 1?



(A) $4\pi\sqrt{3}$ **(B)** 7π **(C)** $\pi(3\sqrt{3}+2)$ **(D)** $10\pi(\sqrt{3}-1)$ **(E)** $\pi(\sqrt{3}+6)$

Problem 17

A child builds towers using identically shaped cubes of different color. How many different towers with a height 8 cubes can the child build with 2 red cubes, 3 blue cubes, and 4 green cubes? (One cube will be left out.)

(A) 24 **(B)** 288 **(C)** 312 **(D)** $1,260$ **(E)** $40,320$

Problem 18

For some positive integer k, the repeating base-k representation of the (base-ten) fraction $\frac{7}{51}$ is

$$0.\overline{23}_k = 0.232323..._k.$$

What is k?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

Problem 19

What is the least possible value of

$$(x+1)(x+2)(x+3)(x+4)+2019,$$

where x is a real number?

(A) 2017 **(B)** 2018 **(C)** 2019 **(D)** 2020 **(E)** 2021

Problem 20

The numbers $1, 2, \ldots, 9$ are randomly placed into the 9 squares of a 3×3 grid. Each square gets one number, and each of the numbers is used once. What is the probability that the sum of the numbers in each row and each column is odd?

(A) $1/21$ **(B)** $1/14$ **(C)** $5/63$ **(D)** $2/21$ **(E)** $1/7$

Problem 21

A sphere with center O has radius 6. A triangle with sides of length $15, 15,$ and 24 is situated in space so that each of its sides is tangent to the sphere. What is the distance between O and the plane determined by the triangle?

(A) $2\sqrt{3}$ **(B)** 4 **(C)** $3\sqrt{2}$ **(D)** $2\sqrt{5}$ **(E)** 5

Problem 22

Real numbers between 0 and 1, inclusive, are chosen in the following manner. A fair coin is flipped. If it lands heads, then it is flipped again and the chosen number is 0 if the second flip is heads and 1 if the second flip is tails. On the other hand, if the first coin flip is tails, then the number is chosen uniformly at random from the closed interval $[0,1]$. Two random numbers x and y are chosen independently in this manner. What is the probability that

$$|x-y|>\frac{1}{2}?$$

(A) $\frac{1}{3}$ **(B)** $\frac{7}{16}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{9}{16}$ **(E)** $\frac{2}{3}$

Problem 23

Travis has to babysit the terrible Thompson triplets. Knowing that they love big numbers, Travis devises a counting game for them. First Tadd will say the number 1, then Todd must say the next two numbers (2 and 3), then Tucker must say the next three numbers (4, 5, 6), then Tadd must say the next four numbers (7, 8, 9, 10), and the process continues to rotate through the three children in order, each saying one more number than the previous child did, until the number $10,000$ is reached. What is the 2019th number said by Tadd?

(A) 5743 **(B)** 5885 **(C)** 5979 **(D)** 6001 **(E)** 6011

Problem 24

Let p, q, and r be the distinct roots of the polynomial $x^3-22x^2+80x-67$. It is given that there exist real numbers A, B, and C such that

$$\frac{1}{s^3-22s^2+80s-67}=\frac{A}{s-p}+\frac{B}{s-q}+\frac{C}{s-r}$$

for all $s \notin \{p,q,r\}$. What is $\frac{1}{A}+\frac{1}{B}+\frac{1}{C}$?

(A) 243 **(B)** 244 **(C)** 245 **(D)** 246 **(E)** 247

Problem 25

For how many integers n between 1 and 50, inclusive, is $\frac{(n^2-1)!}{(n!)^n}$ an integer? (Recall that $0!=1$.)

(A) 31 **(B)** 32 **(C)** 33 **(D)** 34 **(E)** 35

Answer Key

1. C
2. A
3. D
4. B
5. D
6. C
7. C
8. C
9. B
10. C
11. C
12. E
13. D
14. D
15. E
16. A
17. D
18. D
19. B
20. B
21. D
22. B
23. C
24. B
25. D