The Ivy League Education Center

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Middle School Competitive Math Course

Homework Sample

Developed by Dr. Henry Wan

Homework: On average, **at least 2 hours per week**.

We will carefully review and check their homework and correct any mistakes. Based on his/her work, we will provide the student with individualized help and support.

The only way to learn mathematics is by doing mathematics! Homework assignments are a fundamental part of a mathematics course. Homework is also seen as a welcome challenge and an opportunity for further learning. Homework given prior to a lesson can aid in understanding later during class. Homework also provides opportunities for reinforcement of the material learned in class.

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The following steps are important for solving math problems:

FIND OUT

- **Read** the **problem twice**.
- Underline or **highlight key words** and numerical data.
- Decide what the **question** is asking for.
- **You may need to** translate expressions and equations that are written using English words/sentences into mathematical symbols and operations.
- Have you seen a similar problem before?
- If so, how is this problem similar? How is it different?
- What facts do you have?
- **What conditions are given?**
- What do you know that is not stated in the problem?

CHOOSE A STRATEGY

- How have you solved similar problems in the past?
- What strategies do you know?
- Try a strategy that seems as if it will work.
- If it does not, it may lead you to one that will.

SOLVE IT

- Use the *strategy* you selected and work the problem.

LOOK BACK

- Reread the question.
- Did you answer the question asked?
- Is your answer in the correct units?
- Does your answer seem reasonable?
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Specific *strategies* may vary in name. Most, however, fall into these basic categories:

- Compute or Simplify (C)
- Use a Formula (F)
- Make a Model or Diagram (M)
- Make a Table, Chart or List (T)
- Guess, Check & Revise (G)
- Consider a Simpler Case (S)
- Eliminate (E)
- Look for Patterns (P)

The "Show steps" feature allows you to learn basic mathematics on your own, or it can simply be a nice way to check your work!

Mathematics is fun and beautiful. It is an art.

- Try to solve many different types of problems with the same method.
- Try also to solve one problem with different methods.

Have fun.

Think creatively.

Challenge yourself to Solve.

Name: _____ **Date: _____**

The weekly homework problem sets consist of three types of problems:

(1) **A fast/mental calculation problem set**: this will help students establish excellent basic calculation skills and fast problem-solving ability making all areas of math easier and quicker.

(2) **A comprehensive problem set**: this set consists of 25 questions at the AMC 8/MathCounts level taken from the licensed AMC Database. The set covers the following areas: Counting, Number Theory, Logic Reasoning, Probability, Statistics, Algebra, and Geometry.

(3) **An especially designed problem set**: this focuses on the topic presented in an immediately previous lesson, and provides students with the needed review and reinforcement about material learned in class.

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Please solve as many problems as you can. You must show all your work on this sheet.

Part I:

Fast/Mental Calculation Problem Set

Use the efficient strategy, based on the formula $(a+b)(a-b)=a^2-b^2$***, we introduced in the immediately previous class to solve the following problems.***

Example:

$$113\times 109=(111+2)(111-2)=111^2-2^2=12321-4=12317$$

1. $14\times 12=$ _____

2. $19\times 13=$ _____
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3. 34×46 = _____

4. 73×87 = _____

5. 104×116 = _____

6. 113×109 = _____

Bonus:

7. 11115×11117 = _____

Most Challenging Problem:

2015 University of Maryland High School Mathematics Competition Problem 20

Let p be the greatest prime factor of 9,991. The sum of the digits of p is equal to

(A) 4 (B) 10 (C) 13 (D) 16 (E) 28

Hint to get started: $9{,}991 = 10{,}000 - 9 = 100^2 - 3^2$

Part II

Comprehensive Competitive Math Problem Set

Problem 1

$$1-2+3-4+\cdots-2020+2021=$$

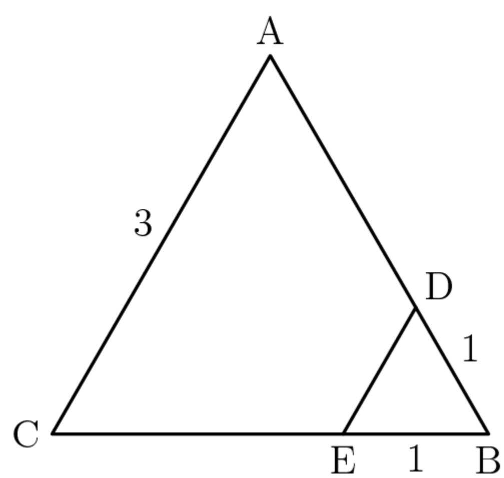
(A) -1011 (B) -1010 (C) 0 (D) 1010 (E) 1011

Problem 2

What is the number halfway between $\frac{1}{16}$ and $\frac{1}{20}$?

(A) $\frac{1}{160}$ (B) $\frac{1}{80}$ (C) $\frac{1}{18}$ (D) $\frac{9}{160}$ (E) $\frac{1}{9}$

Problem 3



A triangular corner with side lengths $\boldsymbol{DB = EB = 1}$ is cut from equilateral triangle $\boldsymbol{ABC}$ of side length 3. What is the perimeter of the remaining quadrilateral?

(A) 6 (B) $6\frac{1}{2}$ (C) 7 (D) $7\frac{1}{2}$ (E) 8

Problem 4

Estimate the time it takes to send 240 blocks of data over a communications channel if each block consists of 512 "chunks" and the channel can transmit 120 chunks per second.

(A) 0.27 seconds (B) 2.7 seconds (C) 27 seconds
(D) 27 minutes (E) 27 hours

Problem 5

If

$$\frac{b}{a} = 3 \text{ and } \frac{c}{b} = 4,$$

what is the ratio of $a + b$ to $b + c$?

(A) $\frac{1}{4}$ (B) $\frac{4}{15}$ (C) $\frac{4}{7}$ (D) $\frac{3}{4}$ (E) $\frac{4}{5}$

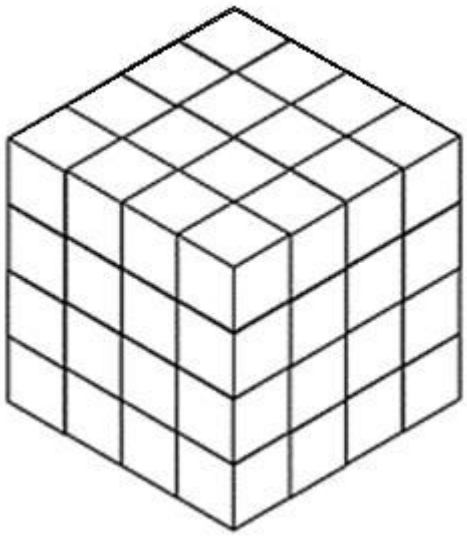
Problem 6

Each day, Alan ate 10% of the jellybeans that were in his bag at the beginning of that day. At the end of the second day, 81 remained. How many jellybeans were in the bag originally?

(A) 100 (B) 105 (C) 110 (D) 115 (E) 125

Problem 7

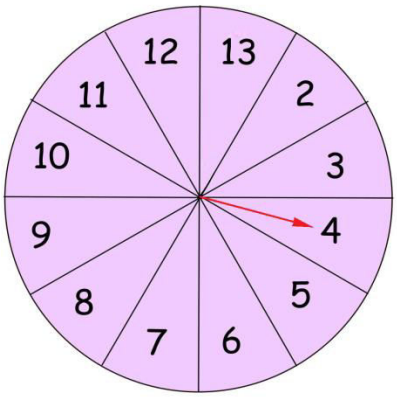
A $4 \times 4 \times 4$ cube is painted blue on the top and the 4 side faces, and red on the bottom face. Then the cube is cut into unit cubes, as shown.



How many of the unit cubes have exactly two blue faces?

(A) 8 (B) 12 (C) 16 (D) 20 (E) 24

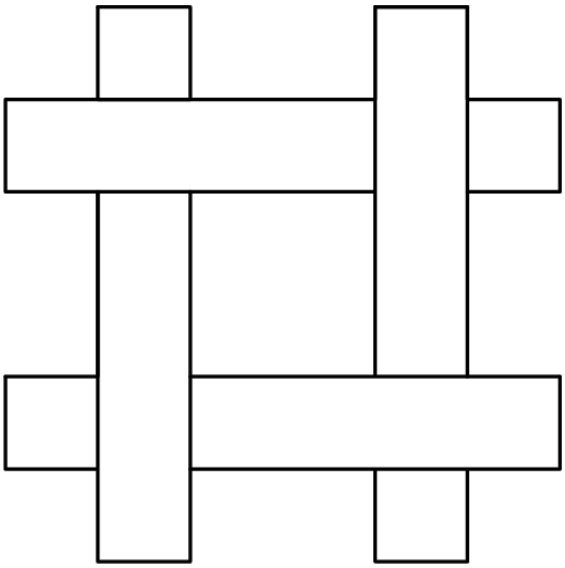
Problem 8



A circular spinner is divided into 12 equal sections, as shown. An arrow is attached to the center of the spinner. The arrow is spun once. What is the probability that the arrow stops in a section containing a prime number that is odd?

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{5}{12}$ (E) $\frac{1}{2}$

Problem 9



Four rectangular paper strips of length 6 and width 1 are put flat on a table and overlap perpendicularly as shown. How much area of the table is covered?

(A) 20 (B) 22 (C) 24 (D) 26 (E) 36

Problem 10

The marked price of a computer was 40% less than the suggested retail price. Joe purchased the book for half the marked price at a Fiftieth Anniversary sale. What percent of the suggested retail price did Joe pay?

(A) 25% (B) 30% (C) 35% (D) 50% (E) 60%

Problem 11

What is the sum of the digits of the decimal form of the product

$$20^{2019} \cdot 50^{2021} ?$$

(A) 2 (B) 4 (C) 7 (D) 8 (E) 10

Problem 12

If $\boldsymbol{a}, \boldsymbol{b}$, and $\boldsymbol{c}$ are digits for which

$$\begin{array}{r} 7\ a\ 2 \\ -\ 4\ 8\ b \\ \hline c\ 7\ 3 \end{array}$$

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

Problem 13

If

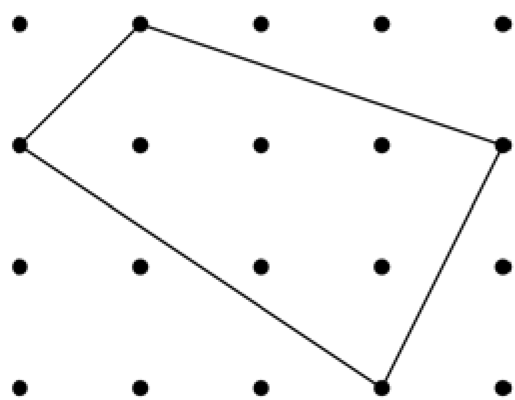
$$\mathbf{3^{2020} - 3^{2021} - 3^{2022} + 3^{2023} = \boldsymbol{m} \cdot 3^{2020}},$$

what is the value of $\boldsymbol{m}$?

(A) 3 (B) 9 (C) 11 (D) 16 (E) 27

Problem 14

Pegs are put in a board 1 unit apart both horizontally and vertically. A rubber band is stretched over 4 pegs as shown in the figure, forming a quadrilateral. What is the area of the quadrilateral in square units?



(A) 4 (B) 4.5 (C) 5 (D) 5.5 (E) 6

Problem 15

When the decimal point of a certain positive decimal number is moved four places to the right, the new number is nine times the reciprocal of the original number. What is the original number?

(A) 0.0003 (B) 0.003 (C) 0.03 (D) 0.3 (E) 3

Problem 16

What is the value of the product

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{2020^2}\right)\left(1-\frac{1}{2021^2}\right)$$

(A) $\frac{1010}{2021}$ (B) $\frac{1}{2}$ (C) $\frac{1011}{2021}$

(D) $\frac{2}{3}$ (E) $\frac{1350}{2021}$

Problem 17

Jim turned his computer off at 5 p.m. Friday, at which point it had been on for exactly 100 hours. At what time had Jim turned his computer on?

(A) 1 p.m. Monday (B) 9 p.m. Monday (C) 1 p.m. Tuesday

(D) 2 p.m. Tuesday (E) 9 p.m. Wednesday

Problem 18

The positive integers $a, b,$ and c have the property that

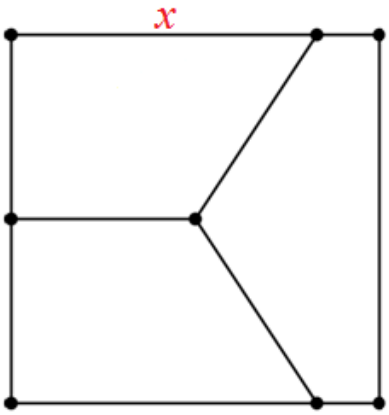
$$abc = 100.$$

What is the smallest possible value of $a + b + c$?

(A) 14 (B) 17 (C) 21 (D) 26 (E) 30

Problem 19

A square with sides of length 1 is divided into two congruent trapezoids and a pentagon, which have equal areas, by joining the center of the square with points on three of the sides, as shown.



Find x, the length of the longer parallel side of each trapezoid.

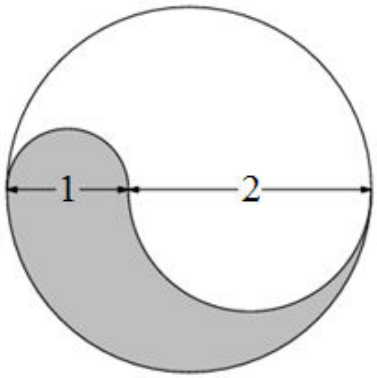
(A) $\frac{3}{5}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{5}{6}$ (E) $\frac{7}{8}$

Problem 20

The mean of three numbers is 20 more than the least of the numbers and 10 less than the greatest. The median of the three numbers is 15. What is their sum?

(A) 15 (B) 25 (C) 30 (D) 40 (E) 45

Problem 21



The figure shown is the union of a circle and two semicircles of diameters 1 and 2, all of whose centers are collinear. What is the ratio of the area, of the unshaded region to that of the shaded region?

(A) $\frac{4}{3}$ (B) $\sqrt{2}$ (C) $\frac{8}{5}$ (D) 2 (E) 4

Problem 22

In how many ways can five different toys be distributed among three children so that each one gets at least one toy?

(A) 120 (B) 150 (C) 180 (D) 210 (E) 240

Problem 23

How many positive integers less than 2021 have an odd number of positive integer divisors?

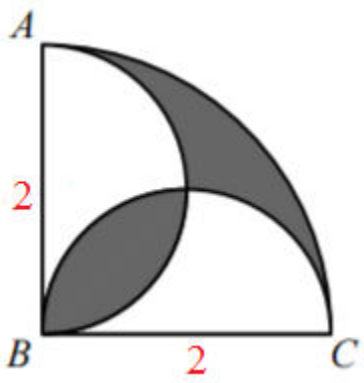
(A) 40 (B) 42 (C) 44 (D) 46 (E) 48

Problem 24

In a class, the ratio of boys to girls is $2:1$. Only $\frac{1}{2}$ of the boys join the math club, whereas $\frac{2}{3}$ of the girls join the math club. A student is selected randomly from the math club. What is the probability that the student selected is a boy?

(A) $\frac{1}{2}$ (B) $\frac{5}{9}$ (C) $\frac{4}{7}$ (D) $\frac{3}{5}$ (E) $\frac{2}{3}$

Problem 25



In the diagram, ABC is a quarter of a circle with radius 8. A semicircle with diameter AB is drawn, as shown. A second semicircle with diameter BC is also drawn. What is the area of the shaded region?

(A) $\frac{3}{4}\pi - \frac{1}{2}$ (B) $\pi - 2$ (C) $\frac{5}{4}\pi - \frac{5}{2}$

(D) $\frac{3}{2}\pi - 3$ (E) $\frac{7}{4}\pi - \frac{7}{2}$