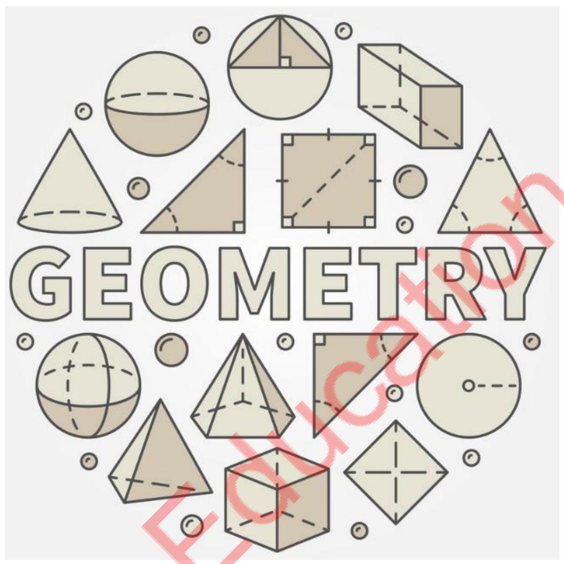
AMC/MathCounts Prep Course

(Tutorial Handout Sample)



***Topic*:** **Angles**

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Contents

1. Introduction

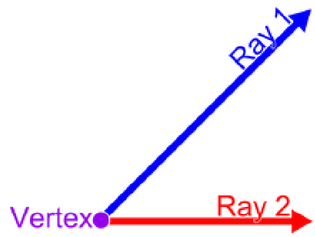
The concept of angle is one of the most important concepts in geometry. Angles are important to defining and studying polygons such as triangles and quadrilaterals. The concepts of equality, sums, and differences of angles are significant and used throughout geometry, but the subject of trigonometry is based on the measurement of angles.

Angles problems frequently occur on the AMC and Mathcounts. This handout shows the importance of angles in competitive math, with an emphasis on angle chasing. Angle chasing is a popular method of solving problems in geometry that employs various properties of, and identities that include angles.

We will use some typical examples to explore several powerful and efficient approaches to solving problems involving angles. In particular, we will go over the skills that are generally lumped under the “angle chasing" category before giving an example as to what angle chasing is. Arithmetic and algebraic methods always play a crucial role in problem-solving in angles.

2. Types of Angles

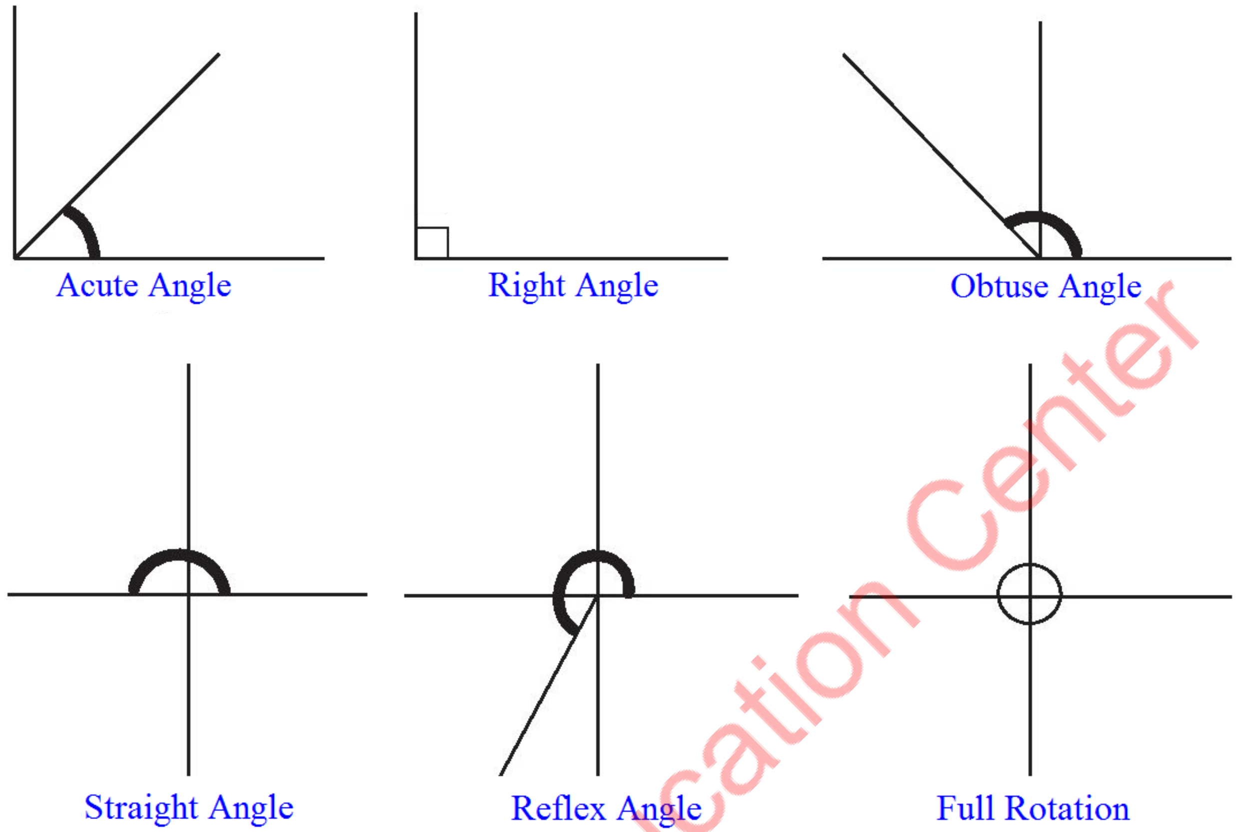
An *angle* is the figure formed by two rays, called the sides of the angle, sharing a common endpoint, called the vertex of the angle. Angles formed by two rays lie in a plane.



Angle is also used to designate the measure of an angle or of a rotation. This measure is the ratio of the length of a circular arc to its radius. In the case of a geometric angle, the arc is centered at the vertex and delimited by the sides. In the case of a rotation, the arc is centered at the center of the rotation and delimited by any other point and its image by the rotation.

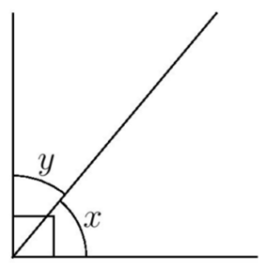
2.1. The Basics

Angle	Classification
$< 90°$	Acute angle
$= 90°$	Right angle
$> 90°$ and $< 180°$	Obtuse angle
$= 180°$	Straight angle
$> 180°$ and $< 360°$	Reflex angle
$= 360°$	Full rotation



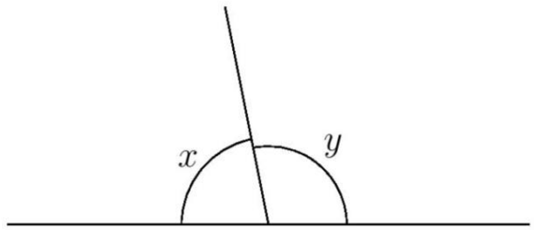
2.2. Complementary Angles

Angles are complementary if they add up to 90°



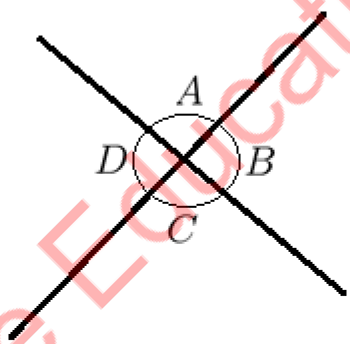
2.3. Supplementary Angles

Angles are supplementary if they add up to 180°



2.4. Opposite Angles (Vertical Angles)

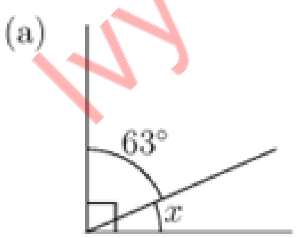
When two lines intersect, they form two pairs of opposite angles, A and C, and B and D. Another word for opposite angles are vertical angles.

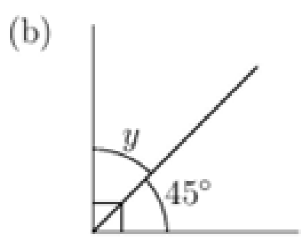


Opposite angles are always congruent, which means that they are equal.

Example 1:

Find the unknown complementary angle.





Solution:

(a)

$$x = 90 - 63° = 27°.$$

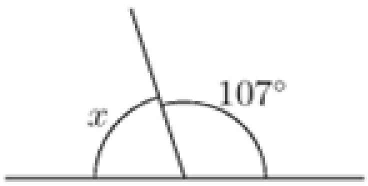
(b)

$$y = 90° - 45° = 45°.$$

Example 2:

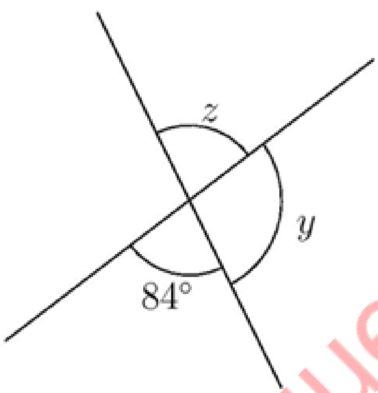
Find the unknown angles.

(a)



(b)

(c)



Solution:

(a)

$$x = 180° - 107° = 73°.$$

(b)

Note that

$$118° + 2y = 180°.$$

Solving for y gives:

$$y = 31°.$$

(c)

By supplementary angles,

$$y = 180° - 84° = 96°.$$

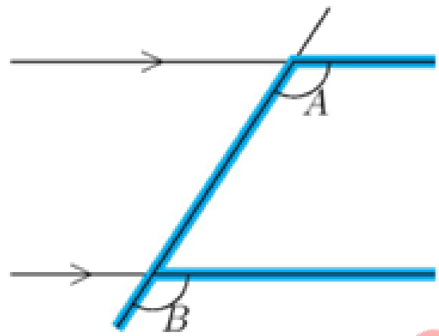
Note that z is the opposite angle to $84°$. So

$$z = 84°.$$

3. Parallel Lines and Angles

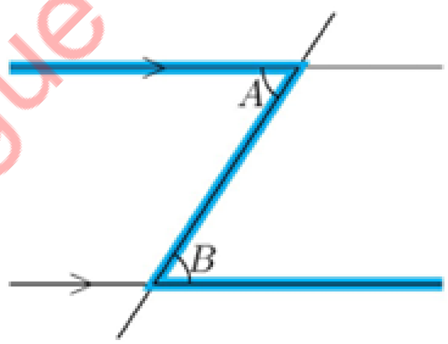
A traversal is a line that intersects two or more lines. If these lines are parallel, the angles around the lines have very nice properties.

3.1. Corresponding Angles



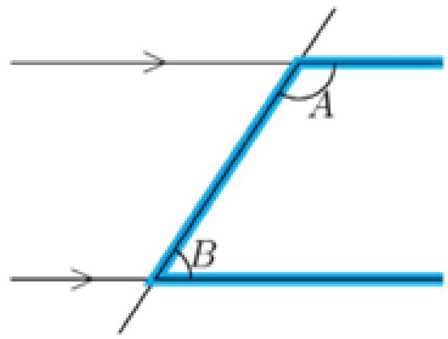
Corresponding angles are angles that are on the same side of the traversal and on the same side of each parallel line. These angles are equal to each other. These types of angles are often referred to as the “F” pattern. The F may also be backward and/or upside-down: ꟻ ⊨ ⅎ

3.2. Alternate Angles



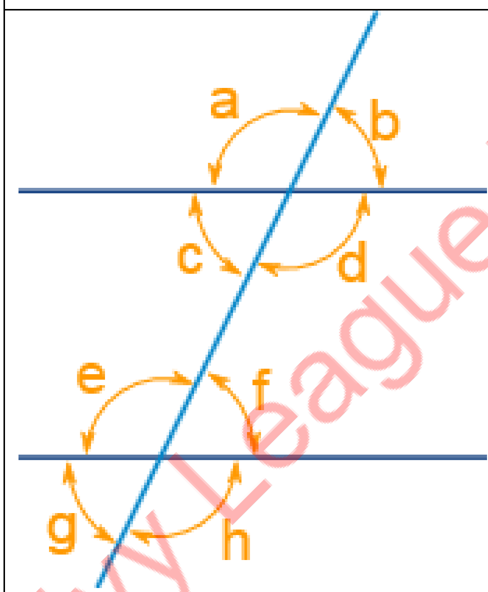
Alternate angles are angles that are on opposite sides of the traversal and on opposite sides of the parallel lines. These angles are equal to each other. These types of angles are often referred to as the "Z" pattern. The Z may also be backward: Ƨ.

3.3. Consecutive Interior Angles



Consecutive angles are angles on the same side of a traversal and on opposite sides of the parallel lines. These angles add up to 180°. This is sometimes referred to as the "C" pattern. The C may also be backward: Ɔ.

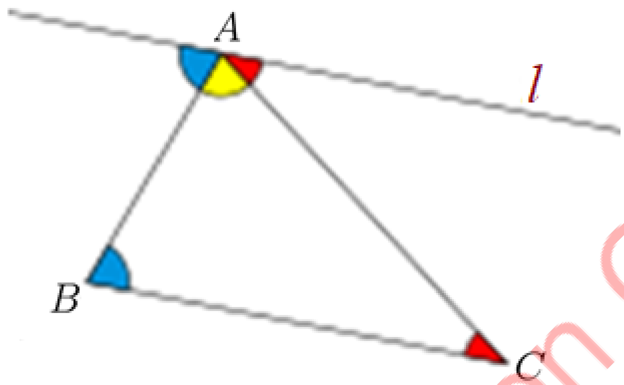
Key Skills: Identify special pairs of angles

Diagram	Property	Example
	*Corresponding Angles* are equal	$\angle a = \angle e$, $\angle b = \angle f$, $\angle c = \angle g$, $\angle d = \angle h$.
	Alternate Interior Angles are equal	$\angle c = \angle f$, $\angle d = \angle e$.
	Consecutive Interior Angles are supplementary	$\angle c + \angle e = 180°$, $\angle d + \angle f = 180°$.

4. Interior and Exterior Angles

4.1. Interior Angles of a Triangle

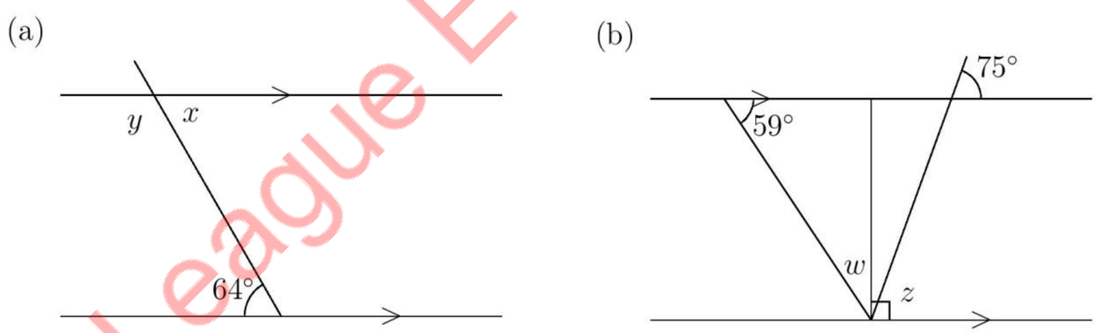
Triangles have three interior angles. These three angles add up to 180°.



Equivalence of the parallel postulate and the "sum of the angles equals to 180°"

Example 3:

Find all the missing angles in the diagrams and state the angle properties you used.



Solution:

(a)

Using alternate angles ("Z" pattern) gives:

$$x = 64°.$$

Using co-interior angles ("C" pattern) yields:

$$y + 64° = 180°.$$

So

$$y = 116°.$$

(b)

By corresponding angles ("F" pattern),

$$z = 75°.$$

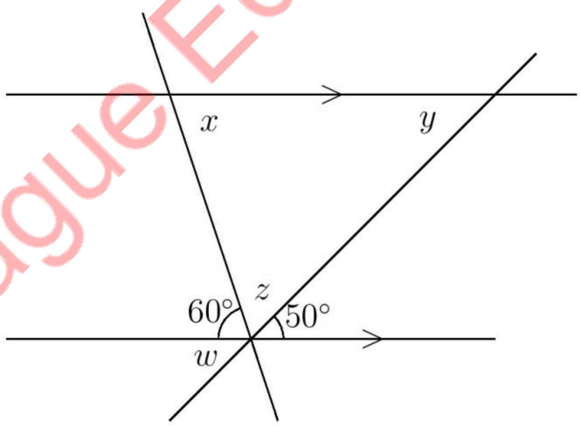
Then using complementary angles, the angle between w and z is $15°$.

Recall that the two acute angles of a right triangle are complementary. So

$$w = 90° - 59° = 31°.$$

Example 4:

Find all the missing angles $w, x, y,$ and z in the diagram using angle properties.



Solution:

By opposite angles,

$$w = 50°.$$

Using alternate angles ("Z" pattern) gives:

$$x = 60° \quad \text{and} \quad y = 50°.$$

Using supplementary angles, we have

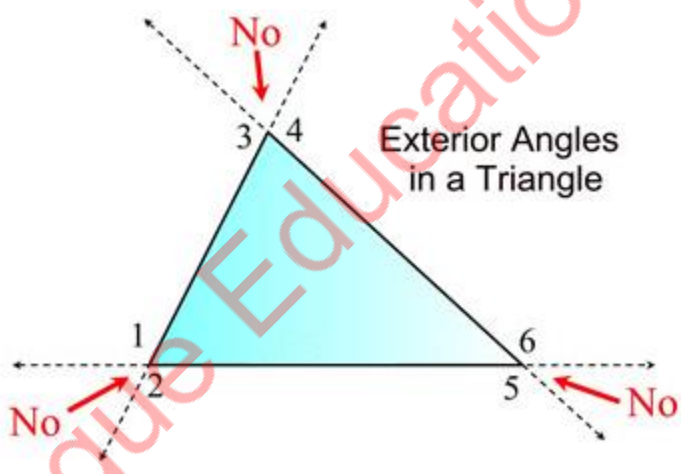
$$60° + z + 50° = 180°.$$

So

$$z = 70°.$$

4.2. Exterior Angles

An exterior angle of a triangle is an angle formed by one side of the triangle and the extension of an adjacent side of the triangle.



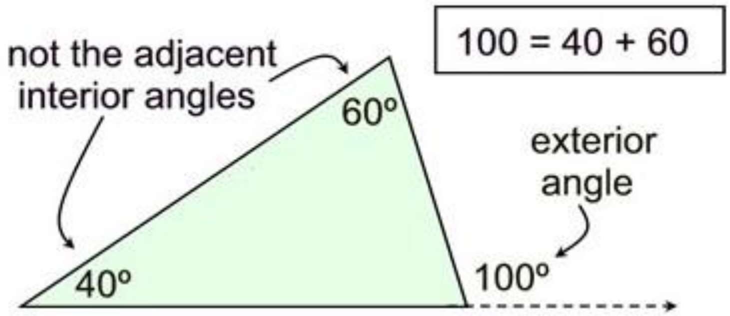
FACTS:

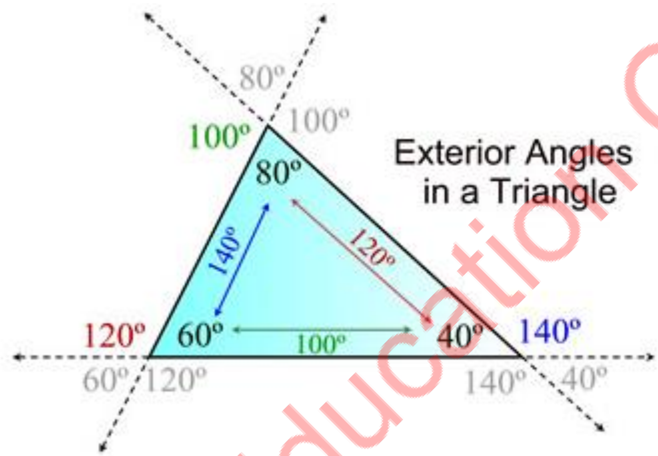
- Every triangle has 6 exterior angles, two at each vertex.
- Angles 1 through 6 are exterior angles.
- Notice that the "outside" angles that are "vertical" to the angles inside the triangle are NOT called exterior angles of a triangle.

Exterior Angle Theorem:

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two non-adjacent interior angles.

(Non-adjacent interior angles may also be referred to as remote interior angles.)





FACTS:

- An exterior angle is equal to the addition of the two interior angles not right next to it.

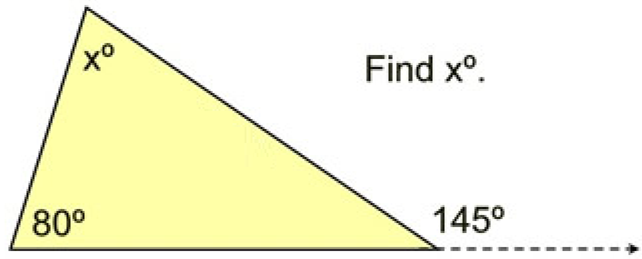
  $140° = 60º + 80°; \qquad 120° = 80° + 40°; \qquad 100° = 60° + 40°.$

- An exterior angle is supplementary to its adjacent interior angle.

  $140°$ is supp to $40°$.

- The 2 exterior angles at each vertex are equal in measure because they are vertical angles.
- The exterior angles (taken one at a vertex) always total $360°$.

Example 5:



Solution:

Using the Exterior Angle Theorem,

$$145 = 80 + x.$$

Solving gives:

$$x = 65.$$

If you forget the Exterior Angle Theorem, you can still get the answer by noticing that a straight angle has been formed at the vertex of the 145° angle.

Solution 2:

The interior angle adjacent to 145° forms a straight angle along with 145° adding to 180°. That interior angle is 35°.

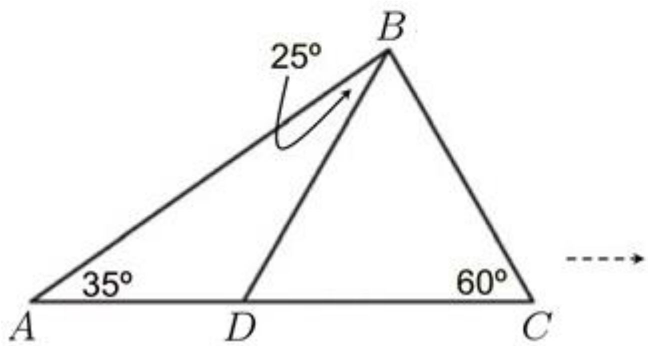
Now use rule that sum of the three interior angles is 180°. We have:

$$35 + 80 + x = 180.$$

Solving gives:

$$x = 65.$$

Example 6:



Find $\angle DBC$.

Solution:

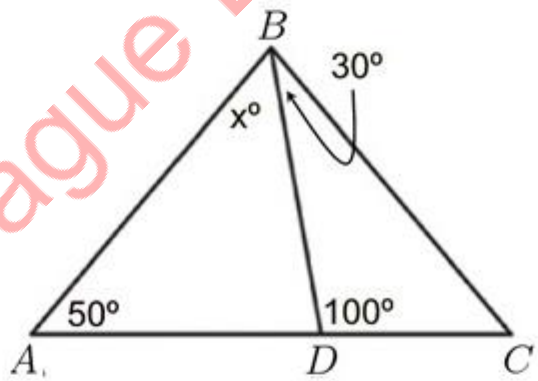
Note that $\angle BDC$ is an exterior angle for ΔABD. Using the Exterior Angle Theorem, we have:

$$\angle BDC = 35° + 25° = 60°.$$

Hence,

$$\angle DBC = 180° - 60° - 60° = 60°.$$

Example 7:



Find x.

Solution:

By the Exterior Angle Theorem, we have:

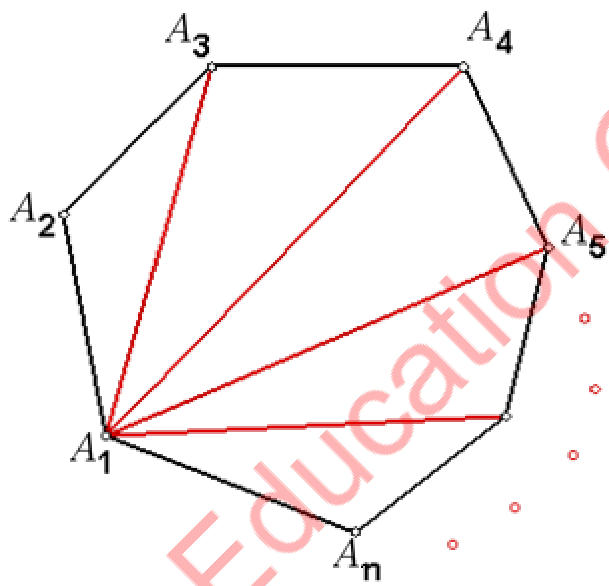
$$100 = x + 50.$$

Hence,

$$x = 50.$$

4.3. Polygons

From any one of the vertices of an n-sided polygon, say A_1, construct diagonals to other vertices.



There are altogether $(n-2)$ triangles.

The sum of angles of each triangle is $180°$.

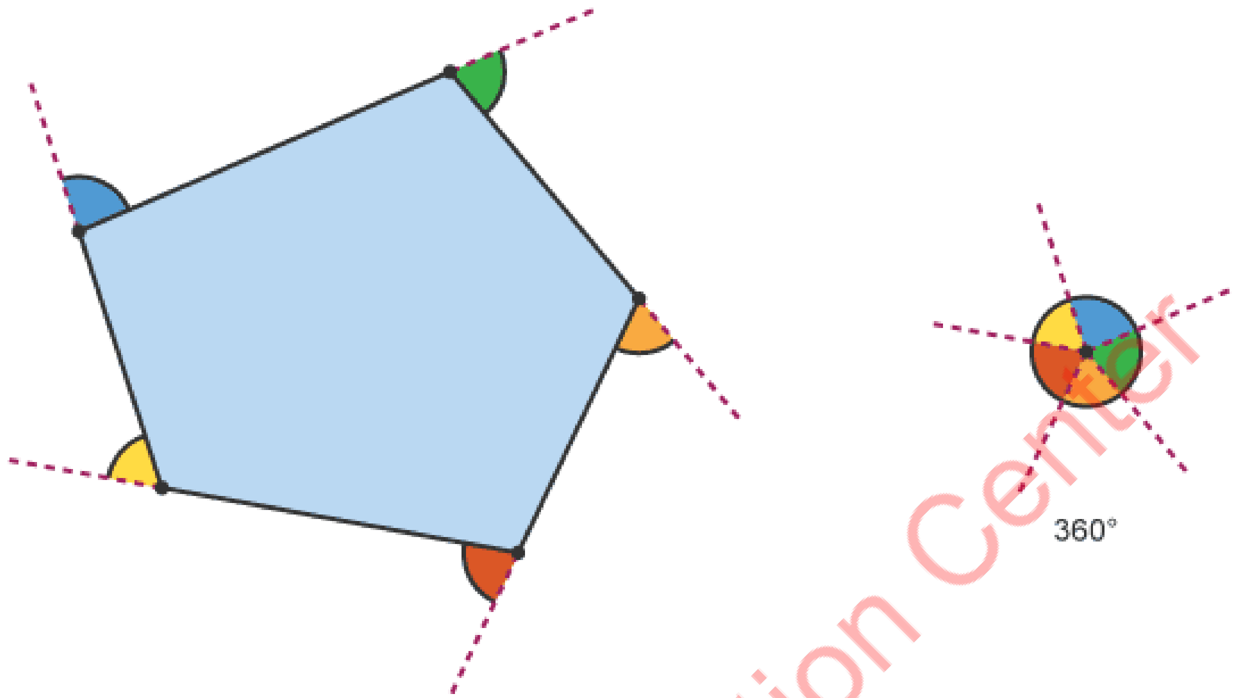
The sum of interior angles of n-sided polygon is

$$(n-2)180°.$$

If the side of a polygon is extended, the angle formed outside the polygon is the *exterior angle*.

The sum of the exterior angles of a polygon is $360°$.

For example, external angles produced along the sides of a pentagon equal 360 degrees



4.4. Regular Polygons

All the interior angles in a regular n-sided polygon are equal.

The measure of an interior angle is:

$$\frac{(n-2)180°}{n}.$$

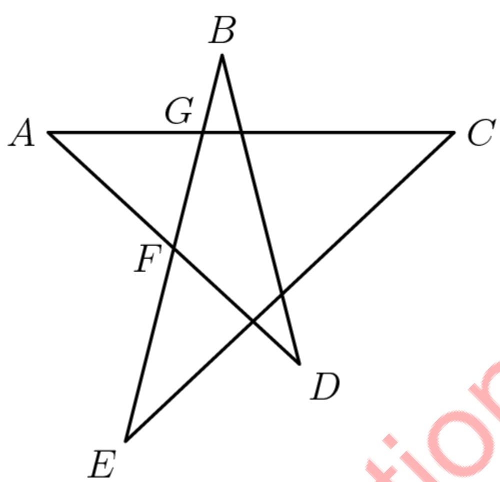
The measure of an exterior angle is:

$$\frac{360°}{n}.$$

5. Problem Solving

Example 5.1: 2000 AMC 8 #24

If $\angle A = 20°$ and $\angle AFG = \angle AGF$, then $\angle B + \angle D =$



(A) $48°$ (B) $60°$ (C) $72°$ (D) $80°$ (E) $90°$

Answer: (D)

Solution:

Note that

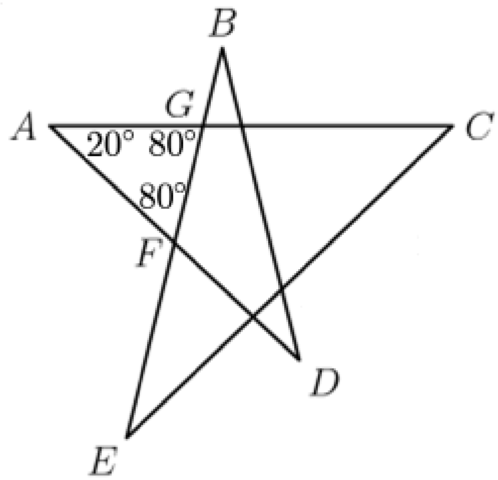
$$\angle AFG = \angle AGF \quad \text{and} \quad \angle GAF + \angle AFG + \angle AGF = 180°.$$

So

$$20° + 2 \cdot \angle AFG = 180°,$$

which implies that

$$\angle AFG = 80°.$$

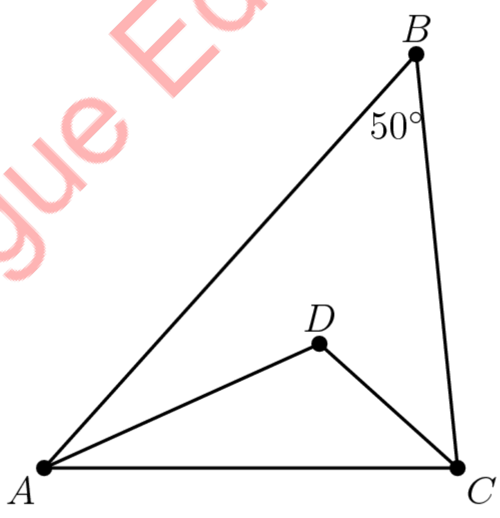


By the Exterior Angle Theorem,

$$\angle B + \angle D = \angle AFG = 80°.$$

Example 5.2: **1996 AJHSME #24**

The measure of angle ABC is $50°$, $\overline{AD}$ bisects angle BAC, and $\overline{DC}$ bisects angle BCA. The measure of angle ADC is



(A) $90°$ (B) $100°$ (C) $115°$ (D) $122.5°$ (E) $125°$

Answer: (C)

Solution:

Recall that the sum of the measures of the angles of a triangle is 180°. So in triangle ABC,

$$\angle BAC + \angle BCA = 180° - 50° = 130°.$$

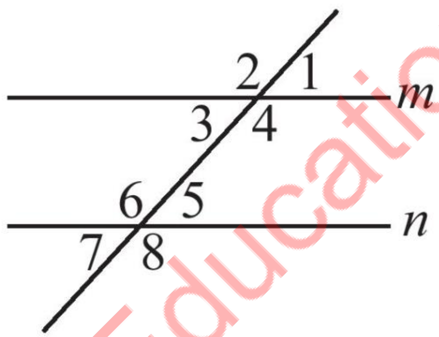
The measures of $\angle DAC$ and $\angle DCA$ are half that $\angle BAC$ and $\angle BCA$, respectively, so

$$\angle DAC + \angle DCA = \frac{\angle BAC + \angle BCA}{2} = \frac{130°}{2} = 65°.$$

Finally, in triangle ACD, we have:

$$\angle ADC = 180° - (\angle DAC + \angle DCA) = 180° - 60° = 115°.$$

Example 5.3: 2021 MathCounts Chapter Sprint #27



In the figure shown, lines m and n are parallel, and the angle labeled 4 measures 104 degrees. What is the degree measure of the angle labeled 7?

Answer: 76

Solution 1

Recall that the *corresponding angles* are equal if the transversal intersects two parallel lines. So

$$\angle 8 = \angle 4 = 104°.$$

Since $\angle 7$ and $\angle 8$ are supplementary, it follows that

$$\angle 7 = 180° - \angle 8 = 180° - 104° = 76°.$$

Solution 2

Recall that the *consecutive interior angles* are supplementary if the transversal intersects two parallel lines. So

$$\angle 5 = 180° - \angle 4 = 180° - 104° = 76°.$$

$$\angle 8 = \angle 4 = 104°.$$

Since $\angle 7$ and $\angle 5$ are vertical angles, it follows that

$$\angle 7 = \angle 5 = 76°.$$

Solution 3

Recall that the *alternate exterior angles* are equal if the transversal intersects two parallel lines. So

$$\angle 6 = \angle 4 = 104°.$$

Because $\angle 7$ and $\angle 6$ are supplementary, it follows that

$$\angle 7 = 180° - \angle 6 = 180° - 104° = 76°.$$

Using the properties of the angles created by parallel lines cut by a transversal, we see that the angle labeled 4 and the angle labeled 6 are alternate interior angles, and alternate interior angles are equal.

Because the angle labeled 4 measures 104, it follows that the angle labeled 6 also measures 104 degrees.

Note that the angle labeled 6 and the angle labeled 7 are supplementary angles. Therefore, the measure of the angle labeled 7 is

$$180 - 104 = \mathbf{76}$$

degrees.

Example 5.4: **2020 AMC 10B #4**

The acute angles of a right triangle are $a°$ and $b°$, where $a > b$ and both a and b are prime numbers. What is the least possible value of b?

(A) 2 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 11

Answer: **(D)**

Solution:

Consider in order the first 5 prime numbers that are possible values of b, namemly

$$2,\ 3,\ 5,\ 7,\ \text{and}\ 11.$$

Note that

$$a = 90 - b.$$

Thus,the corresponding values of a are:

$$88,\ 87,\ 85,\ 83,\ \text{and}\ 79.$$

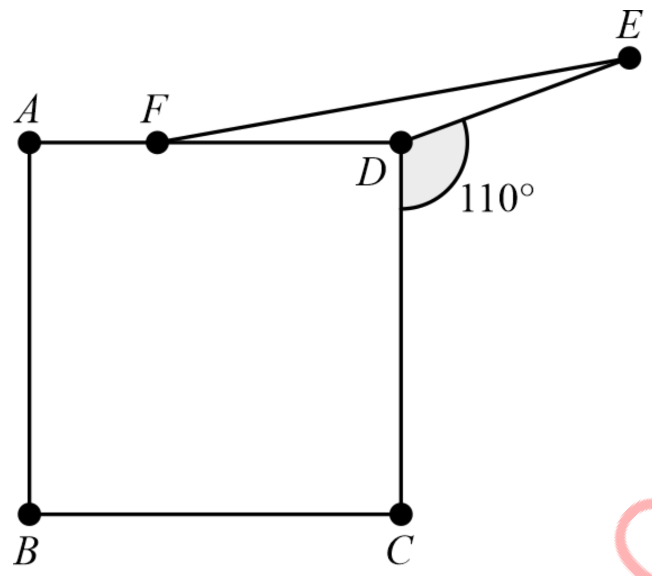
The first prime in this latter list is 83, so

$$b = 7$$

is the least possible value for b for which both a and b are prime and $a > b$.

Example 5.5: **2021 Fall AMC 10A #7/2021 Fall AMC 12A #6**

As shown in the figure below, point E lies in the opposite half-plane determined by line CD from point A so that $\angle CDE = 110°$. Point F lies on $\overline{AD}$ so that $DE = DF$, and $ABCD$ is a square. What is the degree measure of $\angle AFE$?



$(\mathbf{A})\ 160 \qquad (\mathbf{B})\ 164 \qquad (\mathbf{C})\ 166 \qquad (\mathbf{D})\ 170 \qquad (\mathbf{E})\ 174$

Answer: **(D)**

Solution:

By angle subtraction, we have

$$\angle ADE = 360° - \angle ADC - \angle CDE = 360° - 90° - 110° = 160°.$$

Since ΔDEF is isosceles, it follows that

$$\angle DEF = \angle DFE.$$

Note that

$$\angle DEF + \angle DFE + \angle ADE = 180°.$$

So

$$2 \cdot \angle DFE + \angle ADE = 180°,$$

which implies that

$$\angle DFE = \frac{180° - \angle ADE}{2} = \frac{180° - 160°}{2} = 10°.$$

Hence,

$$\angle AFE = 180° - \angle DFE = 180° - 10° = 170°.$$

Example 5.6: 2007 AMC 10A #8/2007 AMC 12A #6

Triangles ABC and ADC are isosceles with

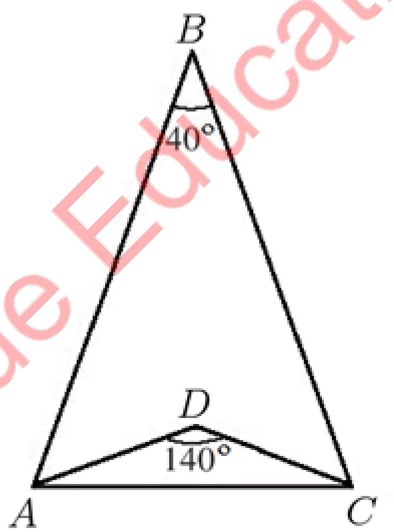
$$AB = BC \text{ and } AD = DC.$$

Point D is inside triangle ABC, angle ABC measures 40 degrees, and angle ADC measures 140 degrees. What is the degree measure of angle BAD?

(A) 20 (B) 30 (C) 40 (D) 50 (E) 60

Answer: **(D)**

Solution 1



Because ΔABC is isosceles, it follows that

$$\angle BAC = \frac{180° - \angle ABC}{2} = \frac{180° - 40°}{2} = 70°.$$

Similarly,

$$\angle DAC = \frac{180° - \angle ADC}{2} = \frac{180° - 140°}{2} = 20°.$$

Hence,

$$\angle BAD = \angle BAC - \angle DAC = 70° - 20° = 50°.$$

Solution 2

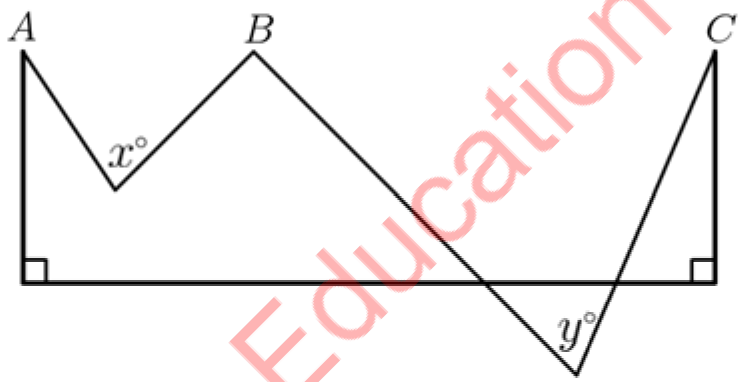
Because ΔABC and ΔADC are isosceles triangles and BD bisects $\angle ABC$ and $\angle ADC$, applying the Exterior Angle Theorem to ΔABD gives

$$\angle BAD + \angle ABD = 70°.$$

Hence,

$$\angle BAD = 70° - 20° = 50°.$$

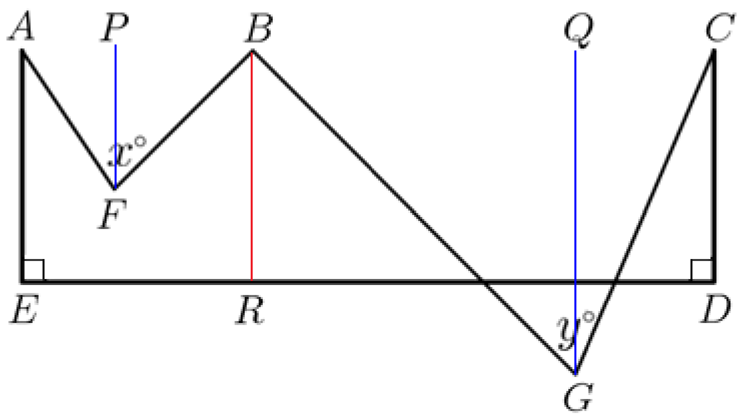
Example 5.7: **2008 MathCounts State Team #9**



In the figure, $m\angle A = 28°$, $m\angle B = 74°$, and $m\angle C = 26°$. If x and y are the measures of the angles in which they are shown, what is the value of $x + y$?

Answer: **128**

Solution:



Label vertices $D,\ E,\ F,$ and $G,$ as shown above. Draw a line parallel to AE through F to meet ED at R. Draw lines PF and QG parallel to AE through F and G, respectively.

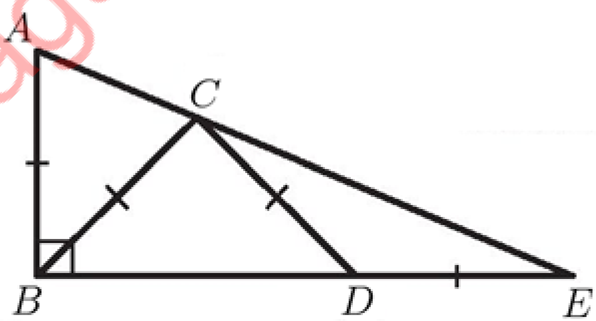
Recall that the alternate interior angles are equal if the transversal intersects two parallel lines. Thus,

$$\angle AFP = \angle A,\ \ \angle BFP = \angle FBR,\ \ \angle BGQ = \angle GBR,\ \ \angle CGQ = \angle C.$$

Adding these 4 equalities together gives:

$$x + y = \angle A + \angle B + \angle C = 28 + 74 + 26 = 128.$$

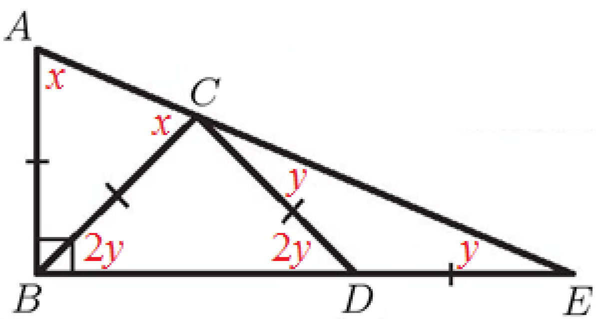
Example 5.8: 2015 MathCounts State Sprint #24



Points C and D are chosen on the sides of right triangle ABE, as shown, such that the four segments $AB, BC, CD,$ and DE each have length 1 inch. What is the measure of angle BAE, in degrees? Express your answer as a decimal to the nearest tenth.

Answer: 67.5

Solution:



Let $\angle BAE = x°$ and $\angle CED = y°$. Since $\angle BAE$ and $\angle CED$ are complementary, it follows that

$$x + y = 90. \tag{1}$$

$$x + y = 90.$$

Note that ΔCDE is isosceles. So

$$\angle ECD = \angle CED = y.$$

Applying the Exterior Angle Theorem to ΔCDE gives

$$\angle CDB = \angle ECD + \angle CED = y + y = 2y.$$

Since ΔCBD is isosceles, it follows that

$$\angle CBD = \angle CDB = 2y.$$

Notice that ΔABC is isosceles. Thus,

$$\angle ABC = 180 - 2 \cdot \angle EAB = 180 - 2x.$$

Because $\angle ABC$ and $\angle CBD$ are complementary, it follows that

$$180 - 2x + 2y = 90.$$

This simplifies to

$$x - y = 45. \tag{2}$$

Adding Equations (1) and (2) together gives:

$$2x = 135.$$

Hence,

$$x = \frac{135}{2} = 67.5.$$

Example 5.9: 2010 AMC 10A #14/2010 AMC 12A # 8

Triangle ABC has $AB = 2 \cdot AC$. Let D and E be on $\overline{AB}$ and $\overline{BC}$, respectively, such that

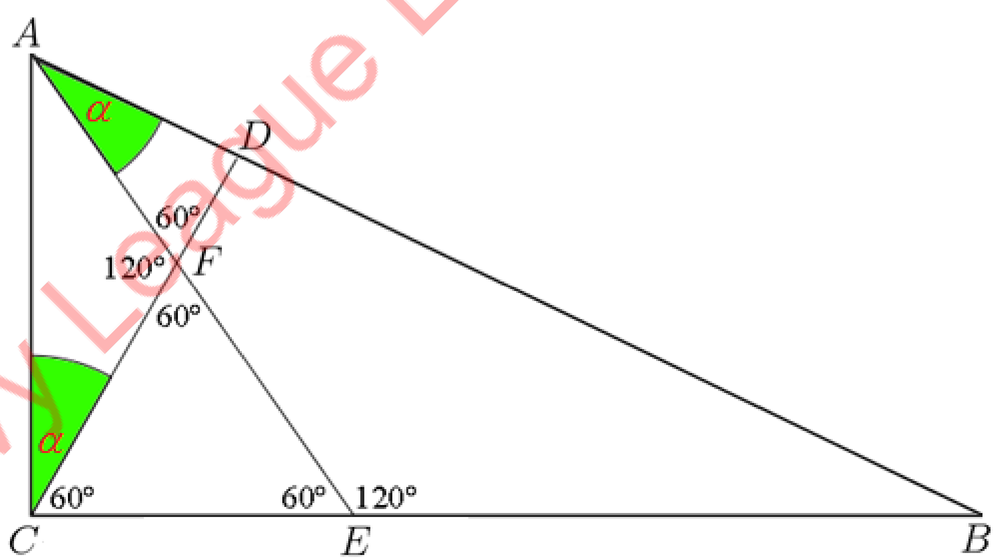
$$\angle BAE = \angle ACD.$$

Let F be the intersection of segments AE and CD, and suppose that ΔCFE is equilateral. What is $\angle ACB$?

(A) $60°$ **(B)** $75°$ **(C)** $90°$ **(D)** $105°$ **(E)** $120°$

Answer: **(C)**

Solution



Let $\alpha = \angle BAE = \angle ACD = \angle ACF$. Because $\angle CFE$ is equilateral, it follows that

$$\angle CFE = 60°.$$

By the Exterior Angle Theorem,

$$\angle FAC + \angle ACF = \angle CFE,$$

so

$$\angle FAC = \angle CFE - \angle ACF = 60° - \alpha$$

and

$$\angle BAC = \angle BAE + \angle FAC = \alpha + (60° - \alpha) = 60°.$$

Since

$$AB = 2 \cdot AC,$$

it follows that ΔBAC is a 30-60-90 triangle, and thus

$$\angle ACB = 90°.$$

CLASSWORK

Exercise Set I

Exercise 1.

Find the degree measure of each of the following angles.

(a) The complementary angle of $57°$.

(b) The supplementary angle of $95°$.

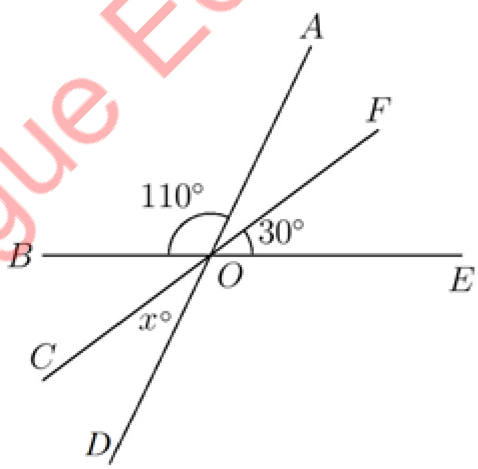
(c) The opposite angle to $112°$.

(d) If angle $w°$ and $x°$ are complementary and $y°$ and $z°$ are complementary, what is the value of $(w + y + z)$?

(e) If $x°$ is supplementary to $42°$ and $y°$ is complementary to $42°$, what is the value of $x + y$?

Exercise 2.

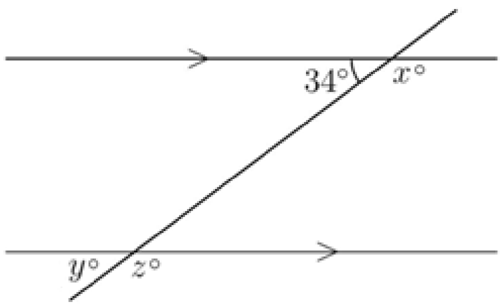
In the diagram, what is the value of x?



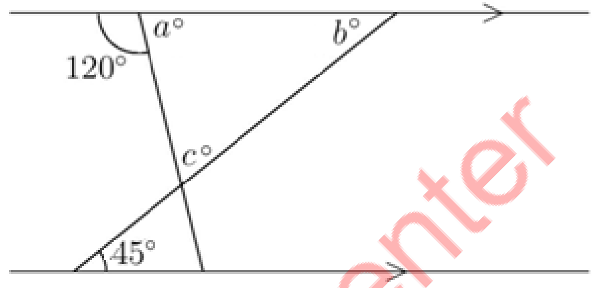
Exercise 3.

Find the degree measure of each of the missing angles in the following diagrams and explain your reasoning.

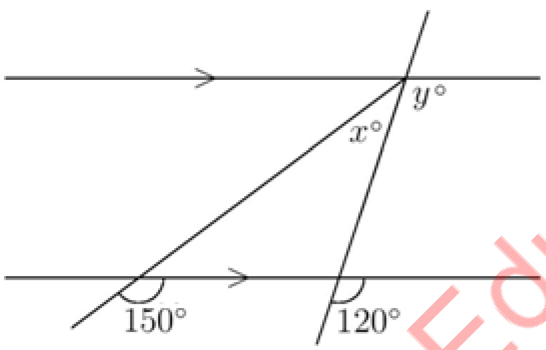
(a)



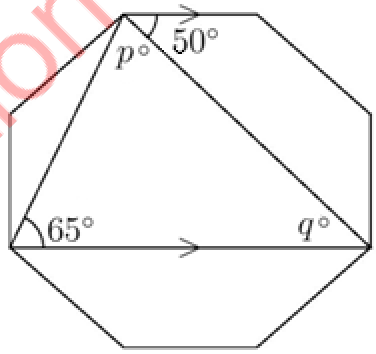
(b)



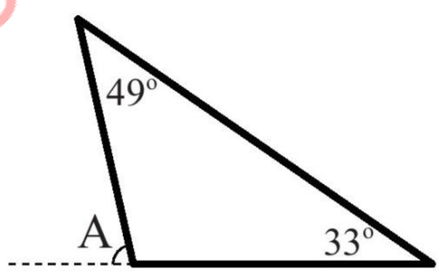
(c)



(d)

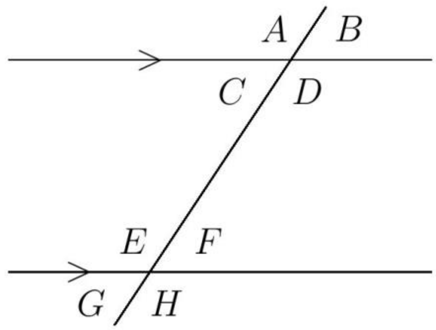


Exercise 4. 2021 MathCounts Chapter Sprint #14



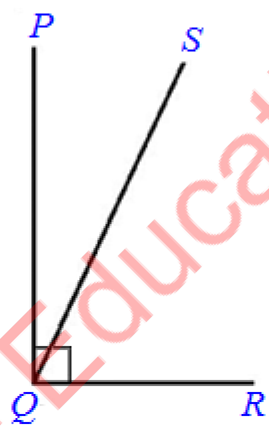
What is the degree measure of angle *A* in the figure shown here?

Exercise 5.



List all pairs of angles that are supplementary to each other in the diagram.

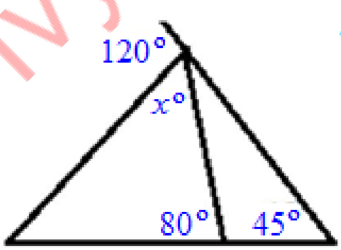
Exercise 6.

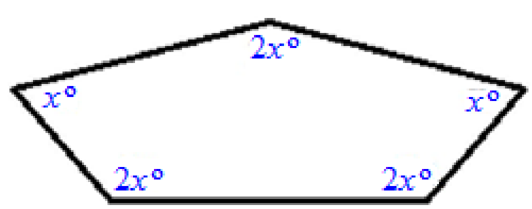


In the diagram, $\angle PQR$ is 90°, and $\angle RQS$ is 50° greater than $\angle PQS$. What is the measure of $\angle PQS$.?

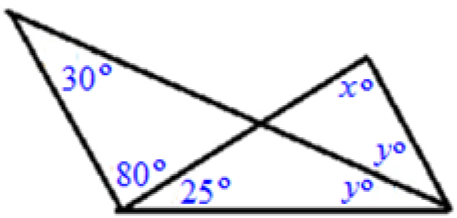
Exercise 7.

Find the degree measure of each of the missing angles.

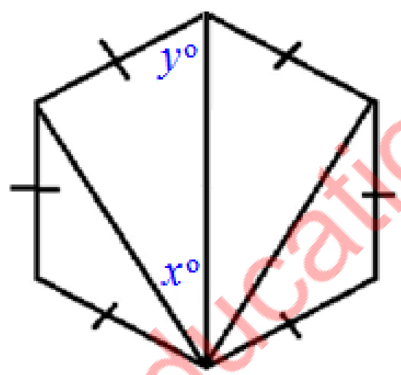
a) 

b) 

c)



Exercise 8.



Find x and y in degrees in the above regular hexagon.

Exercise 9.

Two angles are supplementary to each other. One is 56 degrees larger than the other. What is the degree measure of the smallest angle.

Exercise 10.

Eric ordered a pizza and it was cut into 10 even pieces. What is the degree measure of the angle of a slice?

Exercise 11.

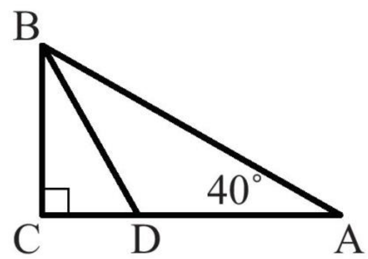
Find a pattern in the sum of angles inside a triangle, square, pentagon . . . Using this pattern What is the sum of interior angles in a 92 sided polygon?

Exercise 12.

The $\angle A$ is three times the size $\angle B$ and $\angle B$ is twice the size of $\angle C$. If $\angle A$ and two times $\angle C$ are supplementary what is $\angle B$?

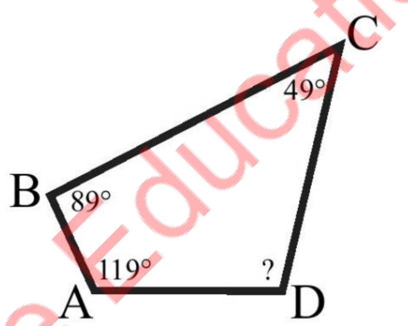
Exercise Set II

Exercise 1. 2020 MathCounts School Sprint #24



In right triangle ABC, shown here, $m\angle A = 40°$, and D is on side AC such that segment BD bisects $\angle CBA$. What is the degree measure of $\angle BDC$?

Exercise 2. 2020 MathCounts Chapter Sprint #7



In quadrilateral $ABCD$, $m\angle A = 119°$, $m\angle B = 89°$, and $m\angle C = 49°$. What is the degree measure of $\angle D$?

Exercise 3. 2020 MathCounts State Sprint #1

In triangle ABC, $m\angle A = 60°$, $m\angle B = 100°$, and $m\angle C = 20°$. If segment BD is constructed inside this triangle so that it bisects $\angle ABC$, what is the degree measure of $\angle BDC$?

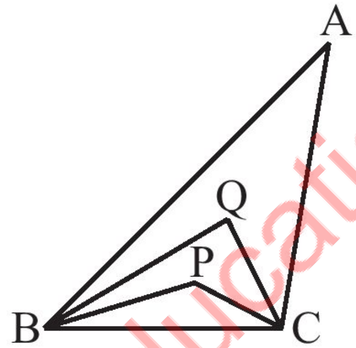
Exercise 4. 2007 MathCounts State Sprint #4

In parallelogram $PQRS$, the measure of angle P is five times the measure of angle Q. What is the measure of angle R, in degrees?

Exercise 5. 2012 MathCounts Chapter Sprint #16

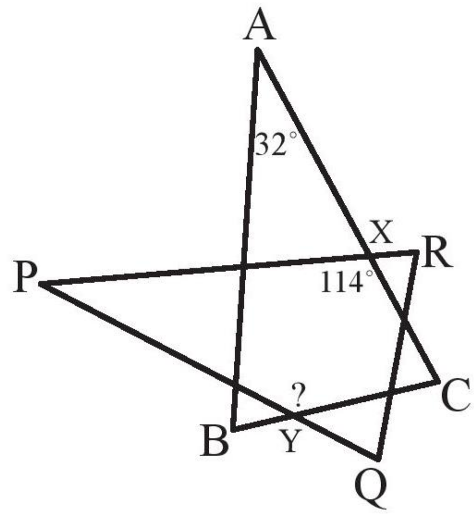
The angles of a triangle are in the ratio $1:3:5$. What is the degree measure of the largest angle in the triangle?

Exercise 6. 2008 MathCounts State Sprint #7



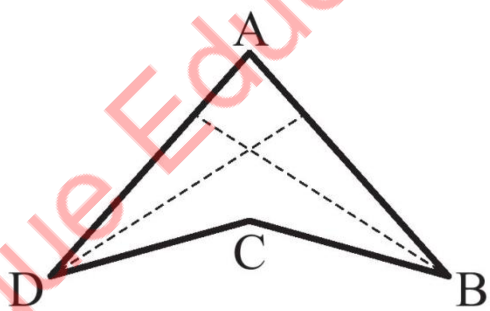
The trisectors of angles B and C of scalene triangle ABC meet at points P and Q, as shown. Angle A measures 39 degrees and angle QBP measures 14 degrees. What is the measure of angle BPC?

Exercise 7. 2016 MathCounts Chapter Sprint #25



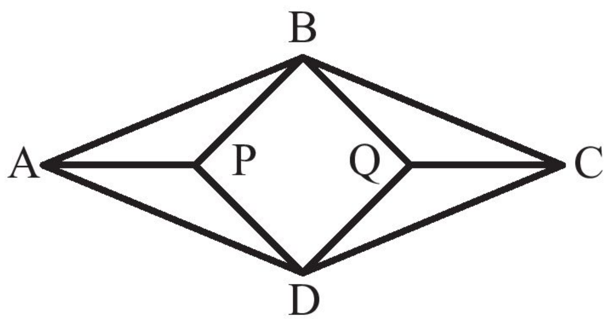
In isosceles triangle ABC, shown here, $AB = AC$ and $m\angle A = 32°$. Triangles ABC and PQR are congruent. Side AC intersects side PR at X so that $m\angle PXC = 114°$, and side PQ intersects side BC at Y as shown. What is the degree measure of $\angle PYC$?

Exercise 8. 2019 MathCounts Chapter Target #4



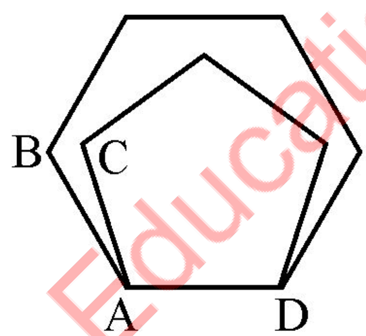
Concave quadrilateral $ABCD$ is symmetric about the line AC. The measures of angles DAB and ABC are 84 degrees and 32 degrees, respectively. The dashed line segments bisect angles ABC and ADC. What is the degree measure of the acute angle at which the two dashed line segments intersect?

Exercise 9. 2017 MathCounts School Target #7



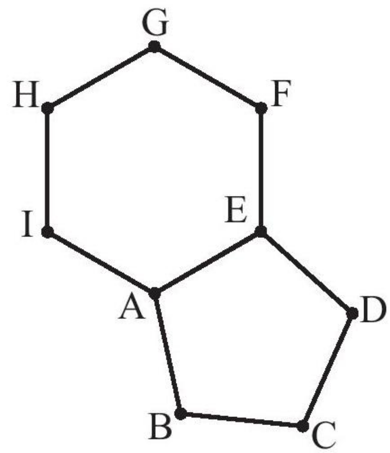
The figure shows points P and Q inside rhombus $ABCD$ so that segments AP, BP, BQ, CQ, DQ, and DP are all congruent. If the measure of angle BAD is 40°, what is the degree measure of angle PDQ?

Exercise 10. 2004 MathCounts Chapter Sprint #27



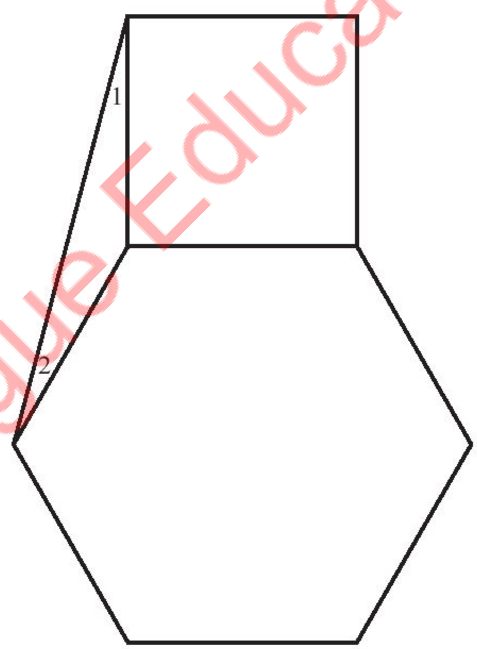
A regular pentagon and a regular hexagon are coplanar and share a common side AD, as shown. What is the degree measure of angle BAC?

Exercise 11. 2010 MathCounts Chapter Sprint #12



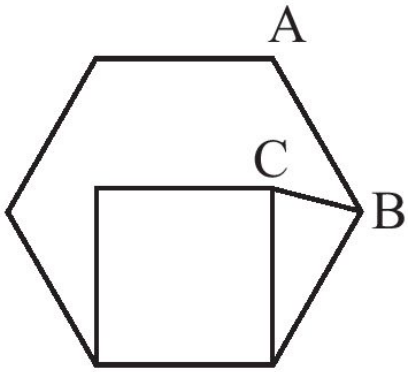
Regular pentagon $ABCDE$ and regular hexagon $AEFGHI$ are drawn on opposite sides of line segment AE such that they are coplanar. What is the degree measure of angle DEF?

Exercise 12. 2013 MathCounts Chapter Team #2



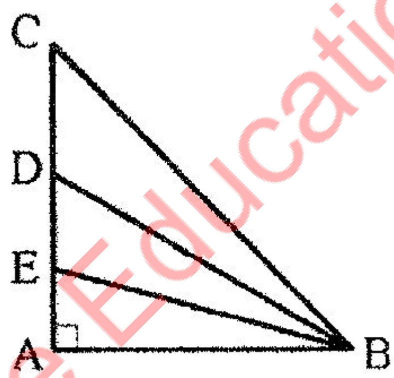
A square and a regular hexagon are coplanar and share a common side as shown. What is the sum of the degree measures of angles 1 and 2?

Exercise 13. 2011 MathCounts State Sprint #19



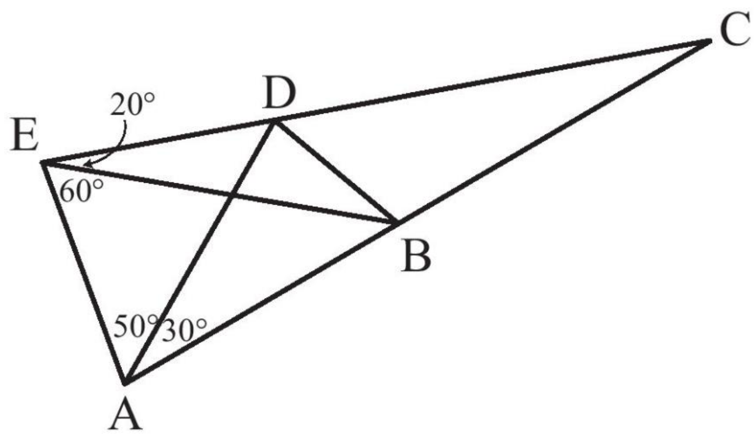
The figure shows a square in the interior of a regular hexagon and sharing a common side. What is the degree measure of $\angle ABC$?

Exercise 14. 2005 MathCounts National Sprint #9



Triangle ABC is an isosceles right triangle with a right angle at A. Segments BD and BE trisect angle ABC. What is the degree measure of angle BDE?

Exercise 15. 2014 MathCounts National Team #10



In the triangle shown, what is the degree measure of $\angle ADB$?

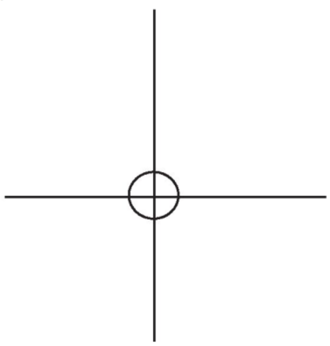
HOMEWORK

Problem Set I

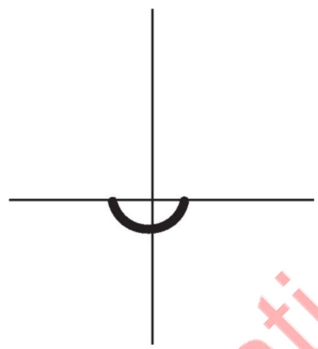
Problem 1.

Classify the angle.

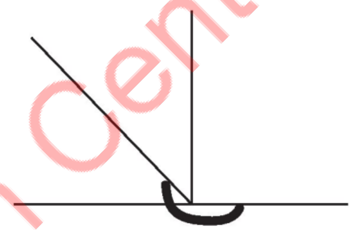
(1)



(2)

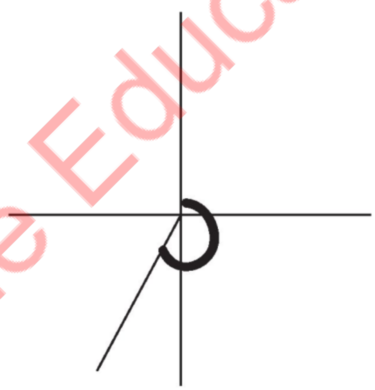


(3)

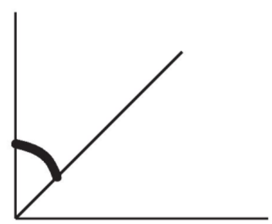


(4)

(5)

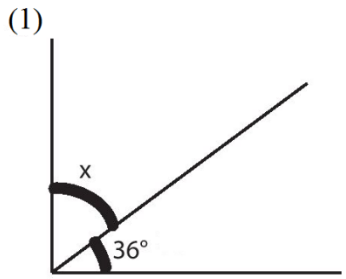


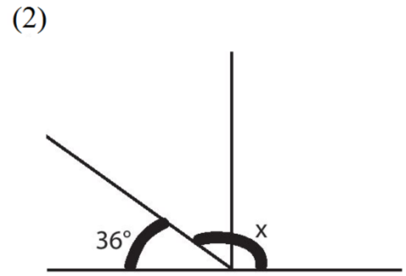
(6)

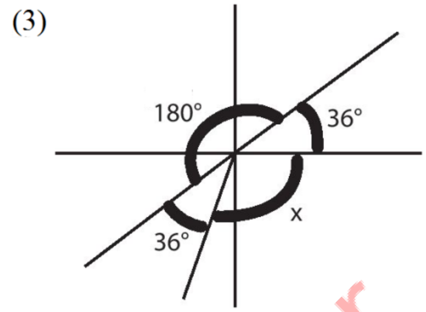


Problem 2.

Find the missing angle, x, in each diagram. (Note: Diagrams may not be to scale)

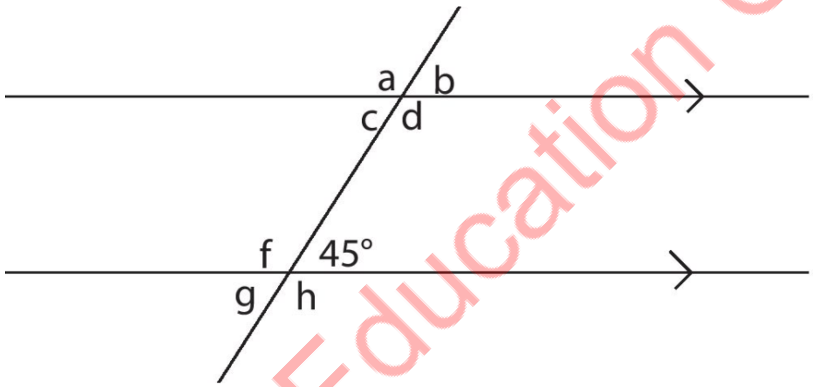






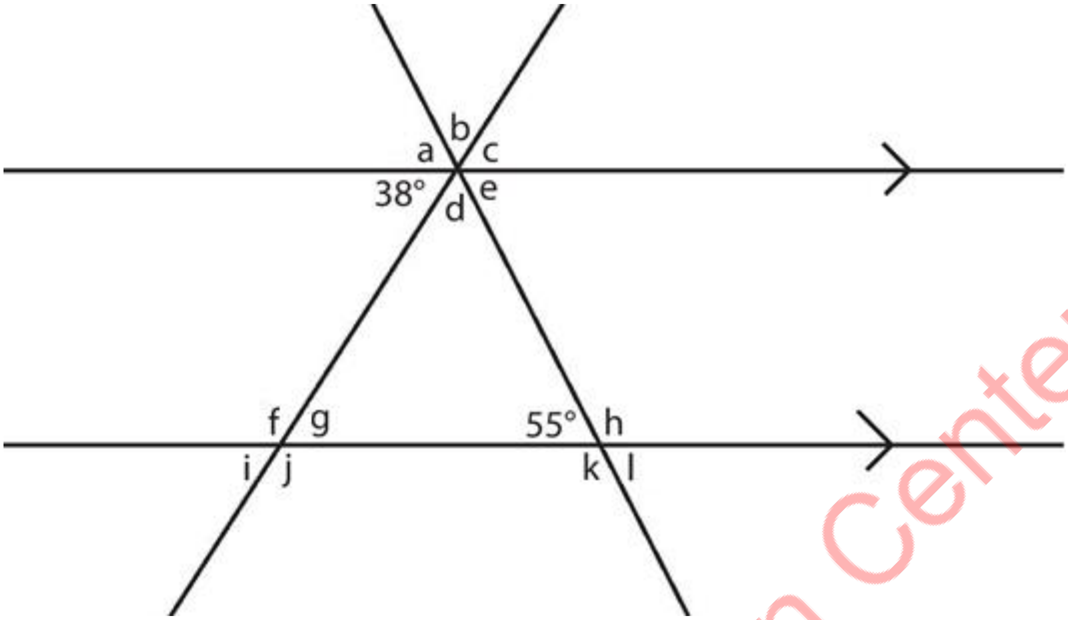
Problem 3.

Find angles $a,\ b,\ c,\ d,\ f,\ g,$ and h.



Problem 4.

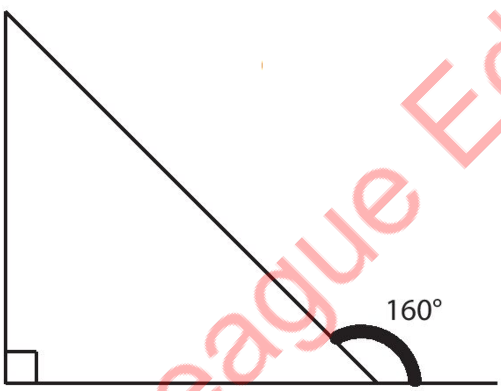
Find angles $a,\ b,\ c,\ d,\ e,\ f,\ g,\ h,\ i,\ j,\ k,$ and l.



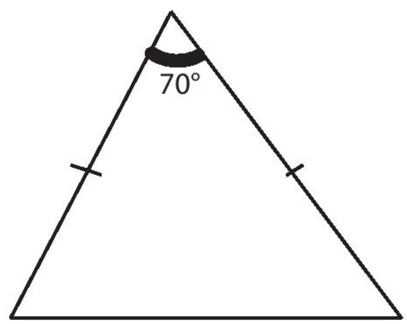
Problem 5.

Classify the triangle and find the missing angle(s): (Note: Diagrams may not be to scale)

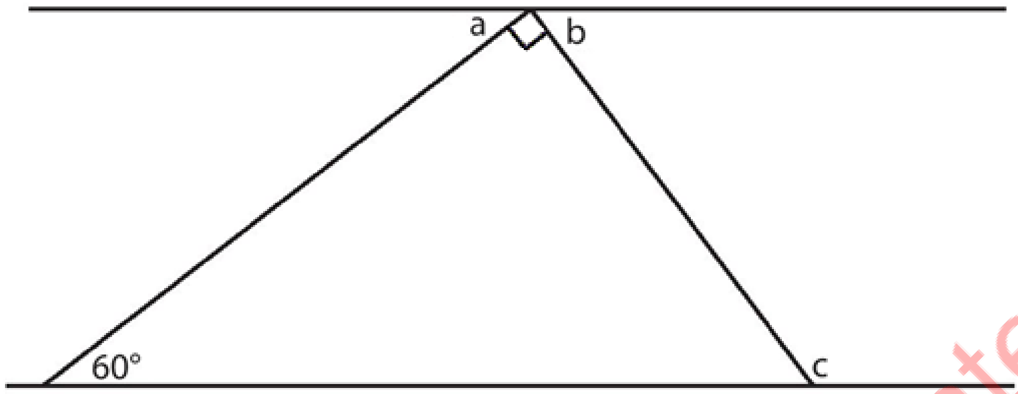
(1)



(2)

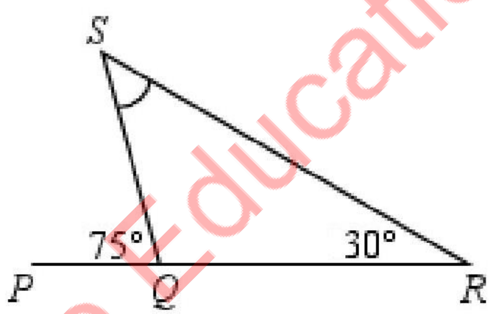


Problem 6.



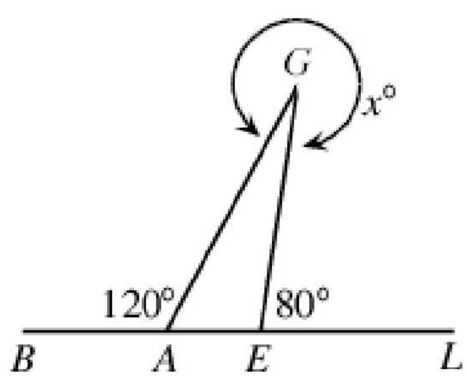
Find angles $a, b,$ and c if the top and bottom lines are parallel.

Problem 7.



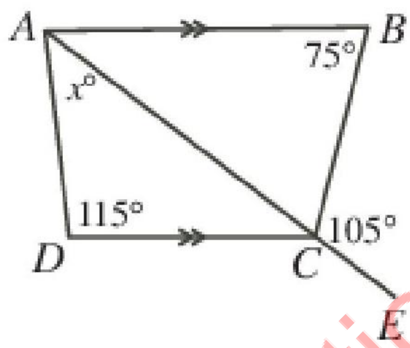
In the diagram shown, PQR is a straight line segment. What is the degree measure of $\angle QSR$?

Problem 8.



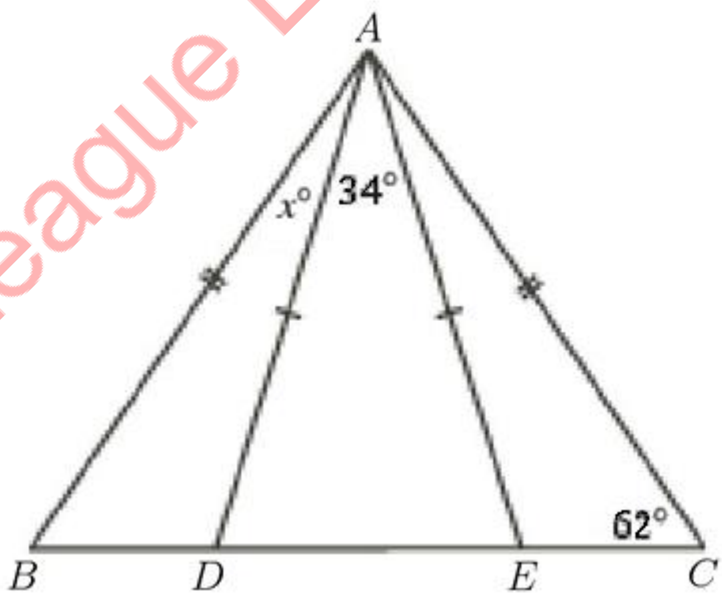
Four points $B, A, E,$ and L are on a straight line as shown. G is a point o the line so that $\angle BAG = 120°$ and $\angle GEL = 80°$. If the reflex angle at G is $x°$, then what does x equal?

Problem 9.



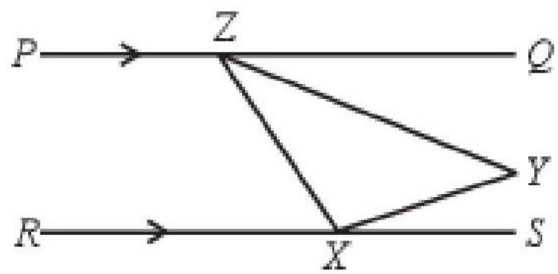
In the diagram, AB is parallel to DC and ACE is a straight line. What is the value of x?

Problem 10.



In the diagram, points D and E lie on BC. Also, $\angle ACB = 62°$, $\angle DAE = 34°$, and $\angle BAD = x°$. What is the value of x?

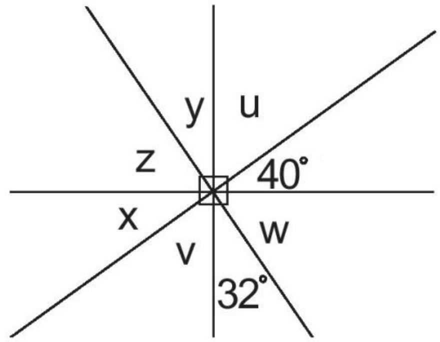
Problem 11.



In the diagram, PQ is parallel to RS. Also, Z is on PQ and X is on RS. If Y is located between PQ and RS so that $\angle YXS = 20°$ and $\angle ZYX = 50°$, what is the measure of $\angle QZY$?

Problem Set II

Problem 1.



In the diagram find the degree measure of each of all the angles.

Problem 2.

What is the angle the hour and minute hand of a clock make when the time is 12:20 p.m.? (Assume the hour hand stays at the hour.)

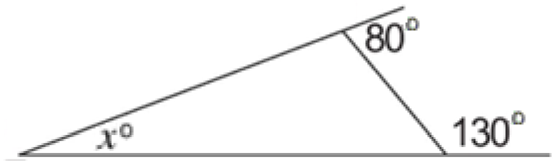
Problem 3.

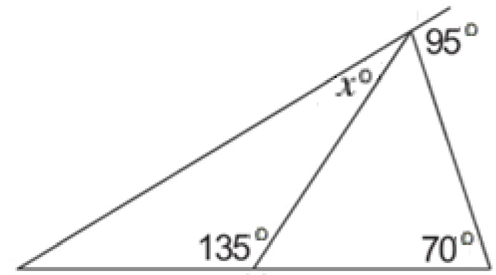
Which of the following set of angles would form an isosceles triangle?

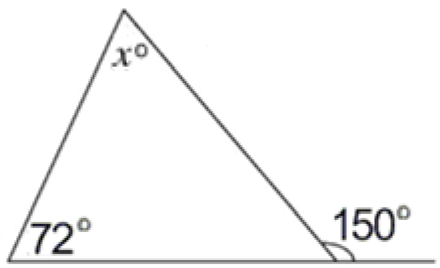
1) $30°,\ 60°,\ 90°$.
2) $50°,\ 30°,\ 100°$.
3) $15°,\ 150°,\ 15°$.
4) $10°,\ 75°,\ 95°$.
5) $20°,\ 20°,\ 140°$.

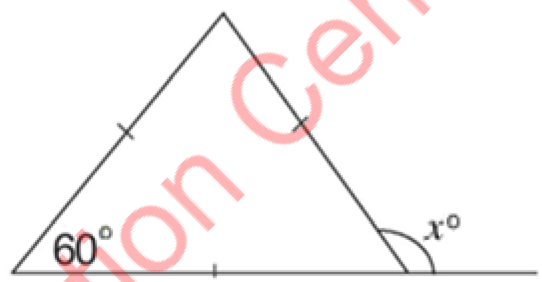
Problem 4.

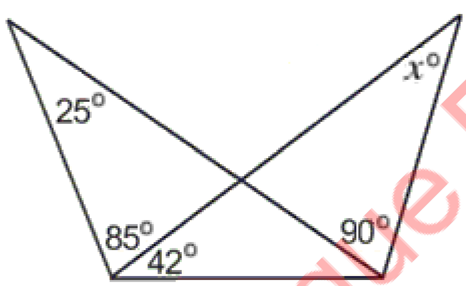
Find the value of x in each of the following figures.

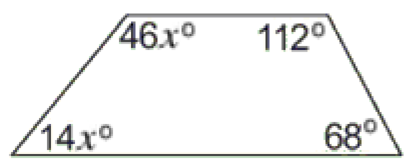
a) 

b) 

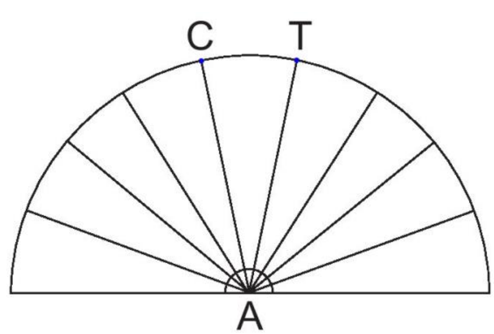
c) 

d) 

e) 

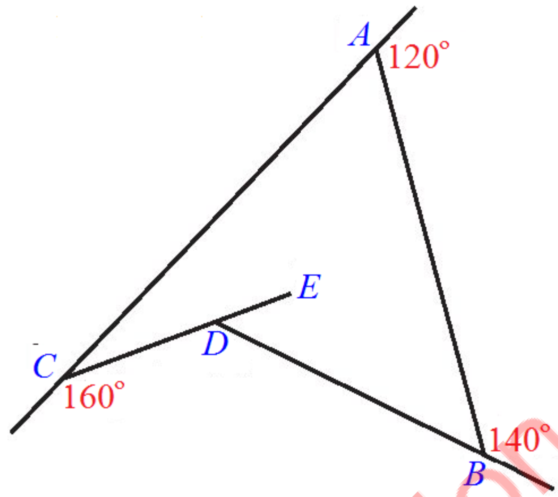
f) 

Problem 5.



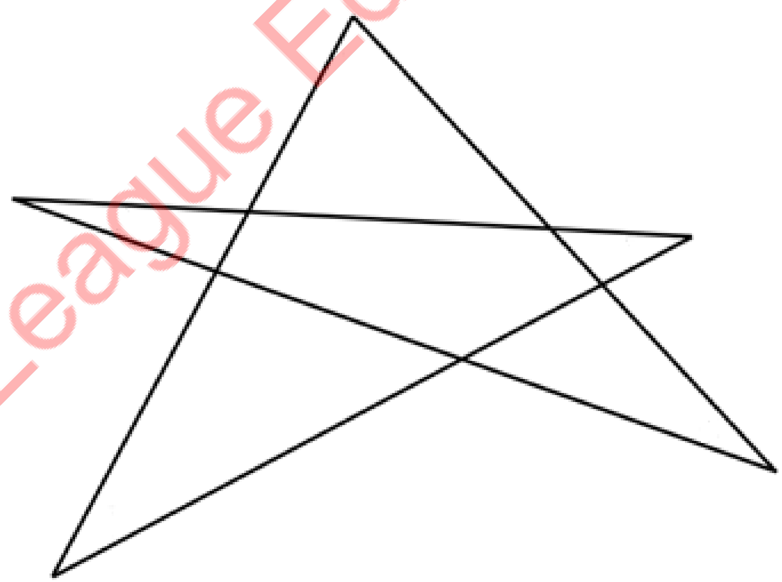
A semicircle is split into 9 equal parts. Find the degree measure of $\angle CAT$.

Problem 6.



In the diagram, what is degree measure of $\angle EDB$?

Problem 7.



What is the sum of the angles at the vertices of a five-pointed star?

Problem 8.

What is the sum of interior angles of a decagon?

Problem 9.

What is the sum of interior angles of a 54-gon?

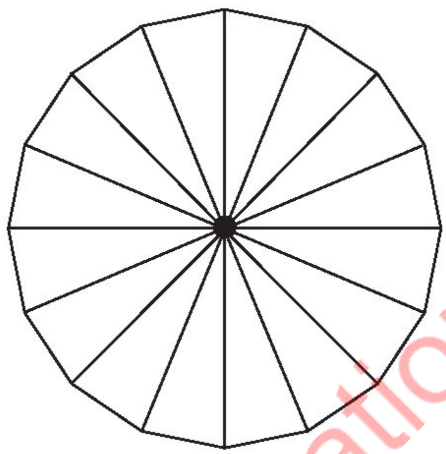
Problem 10.

How many degrees are there in a circle?

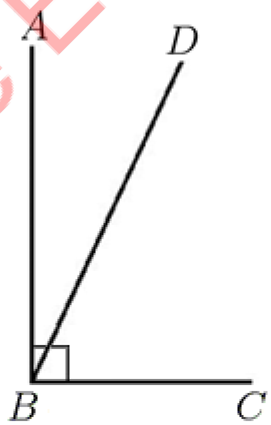
Problem Set III

Problem 1.

There are 16 spokes equally spaced around a wheel. What is the angle between any two spokes in the wheel?

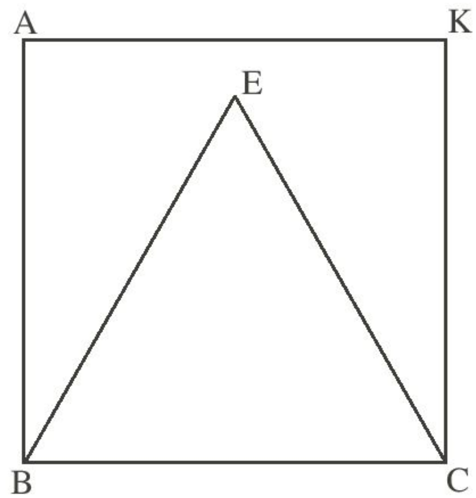


Problem 2.



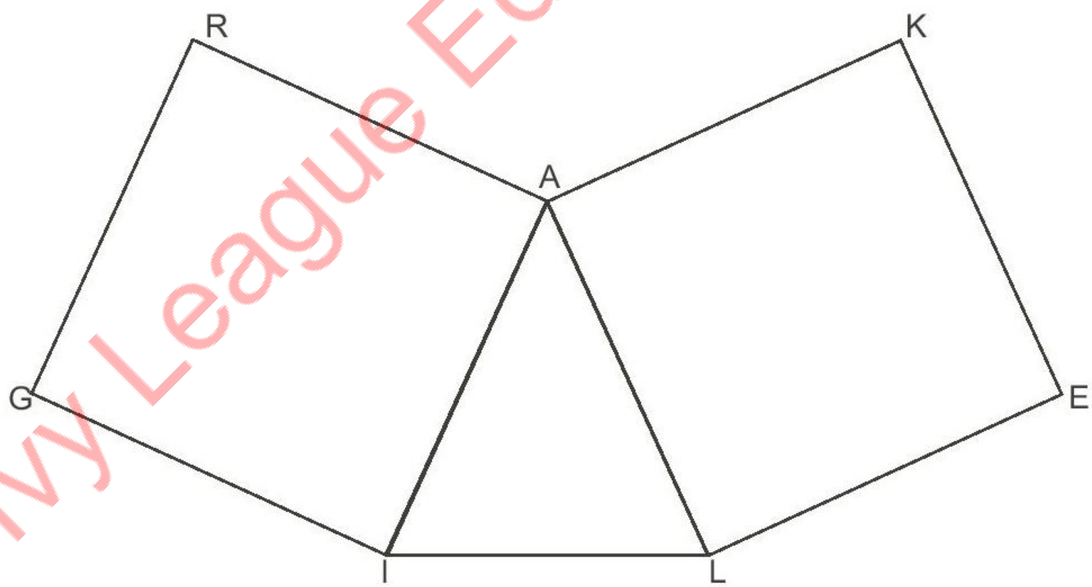
In the diagram, $\angle ABC$ is $90°$. $\angle CBD$ is $30°$ larger than $\angle ABD$. What is the degree measure of $\angle CBD$?

Problem 3.



In the diagram shown above, $BAKC$ is a square and ΔBEC is an equilateral triangle. What is the degree measure of $\angle AEK$?

Problem 4.



In the diagram shown above, $\angle AIL$ is $70°$ and $GRAI$ and $LAKE$ are equal squares. What is the obtuse angle in $\angle RAK$?

Problem 5.

In $\Delta DOG, \angle O$ is 4 degrees larger than $\angle D$ and $\angle G$ is 2 times $\angle D$. What is the degree measure of $\angle D$?

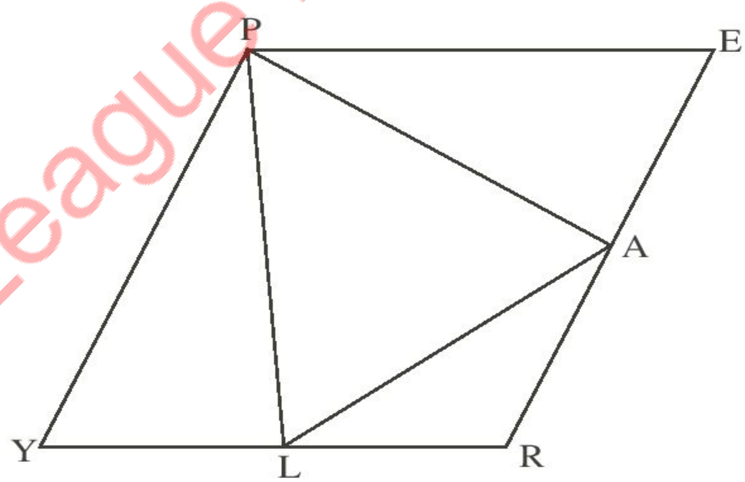
Problem 6.

Two angles are complementary, and one angle is 38° larger than the other one. What is the degree measure of the smaller angle?

Problem 7.

The degree of $\angle A$, is twice the degree of $\angle B$, which is twice the degree of $\angle C$. If $\angle A$ and $\angle C$ are supplementary, what is the degree measure of $\angle B$?

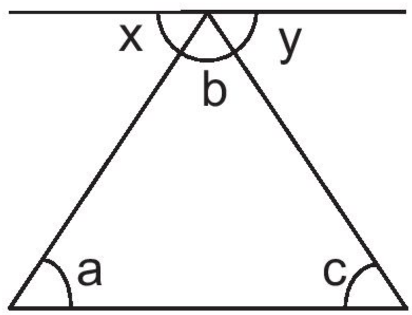
Problem 8.



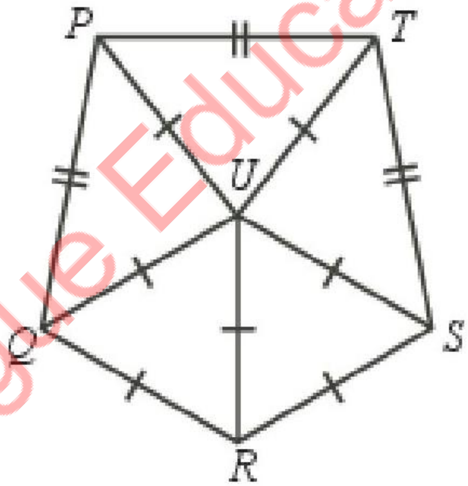
In rhombus $PERY$, points A and L lie on ER and RY, respectively, such that $PE = PA = PL = AL$, what is the degree measure of $\angle EPY$?

Problem 9.

Given the diagram below, can you prove why the sum of all the angles in a triangle equal $180°$?



Problem 10.



In the diagram, ΔQUR and ΔSUR are equilateral triangles. Also, ΔQUP, ΔPUT, and ΔTUS are isosceles triangles with $PU = QU = SU = TU$ and $QP = PT = TS$. What is the measure of $\angle UST$ in degrees?

Problem Set IV

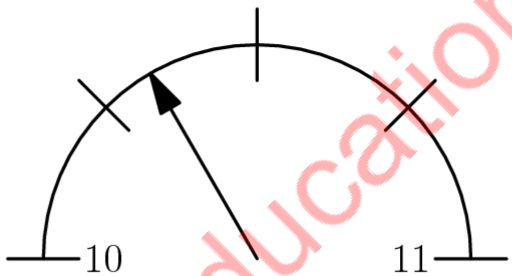
Problem 1: 1987 AJHSME #4

Martians measure angles in clerts. There are 500 clerts in a full circle. How many clerts are there in a right angle?

(A) 90 (B) 100 (C) 125 (D) 180 (E) 250

Problem 2: 1988 AJHSME #1

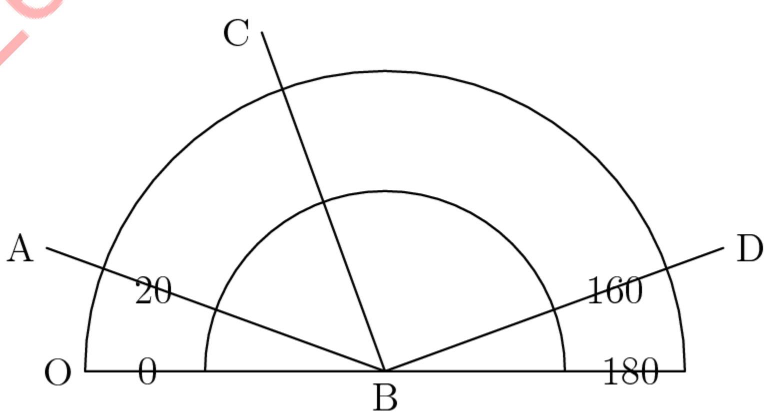
The diagram shows part of a scale of a measuring device. The arrow indicates an approximate reading of



(A) 10.05 (B) 10.15 (C) 10.25 (D) 10.3 (E) 10.6

Problem 3: 1988 AJHSME #5

If $\angle CBD$ is a right angle, then this protractor indicates that the measure of $\angle ABC$ is approximately



(A) 20° (B) 40° (C) 50° (D) 70° (E) 120°

Problem 4: 1989 AJHSME #10

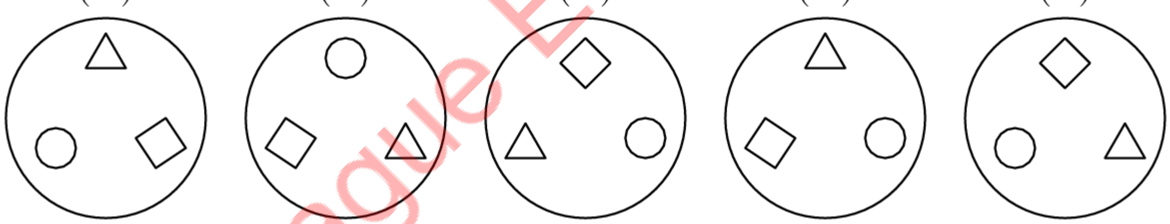
What is the number of degrees in the smaller angle between the hour hand and the minute hand on a clock that reads seven o'clock?

(A) 50° (B) 120° (C) 135° (D) 150° (E) 165°

Problem 5: 1994 AJHSME #4

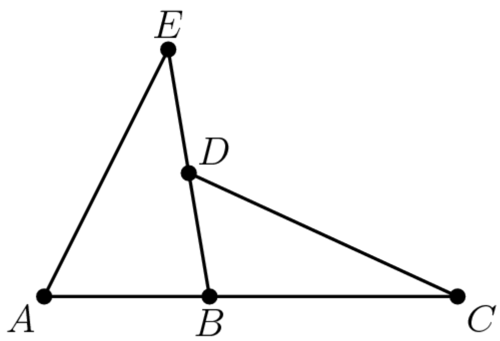
Which of the following represents the result when the figure shown below is rotated clockwise 120° about its center?

(A) (B) (C) (D) (E)



Problem 6: 1994 AJHSME #7

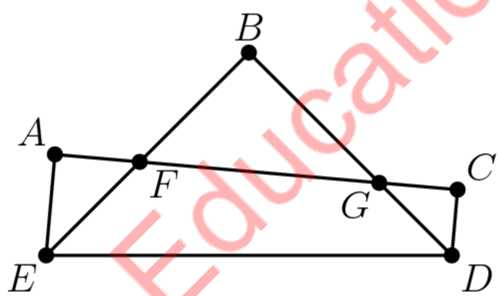
If $\angle A = 60^\circ$, $\angle E = 40^\circ$ and $\angle C = 30^\circ$, then $\angle BDC =$



(A) 40° (B) 50° (C) 60° (D) 70° (E) 80°

Problem 7: 1995 AJHSME #13

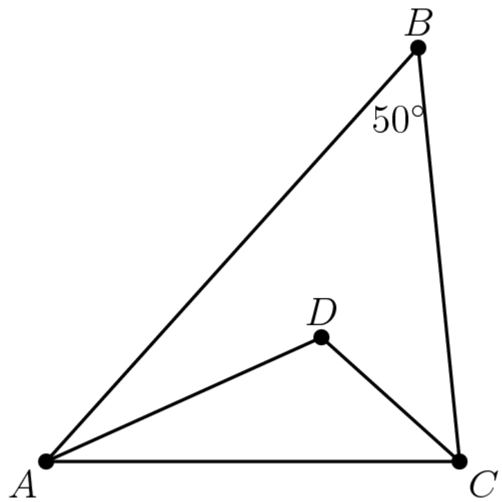
In the figure, $\angle A$, $\angle B$, and $\angle C$ are right angles. If $\angle AEB = 40^\circ$ and $\angle BED = \angle BDE$, then $\angle CDE =$



(A) 75° (B) 80° (C) 85° (D) 90° (E) 95°

Problem 8: 1996 AJHSME #24

The measure of angle ABC is 50°, $\overline{AD}$ bisects angle BAC, and $\overline{DC}$ bisects angle BCA. The measure of angle ADC is



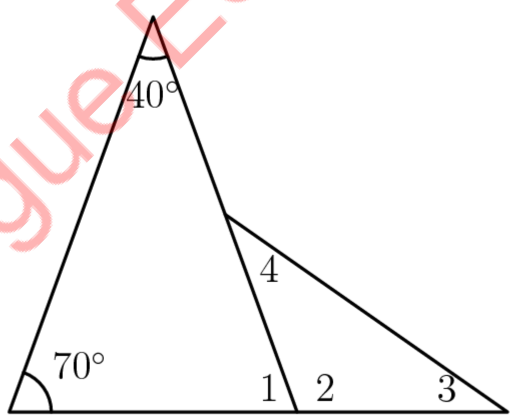
(A) 90° (B) 100° (C) 115° (D) 122.5° (E) 125°

Problem 9: 1997 AJHSME #12

$\angle 1 + \angle 2 = 180°$

$\angle 3 = \angle 4$

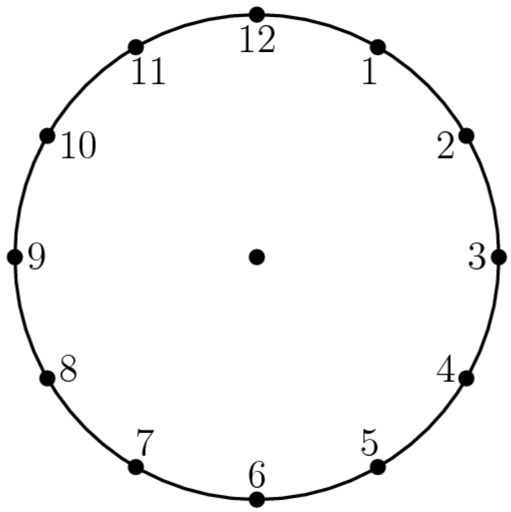
Find $\angle 4$.



(A) 20° (B) 25° (C) 30° (D) 35° (E) 40°

Problem 10: 1999 AMC 8 #2

What is the degree measure of the smaller angle formed by the hands of a clock at 10 o'clock?

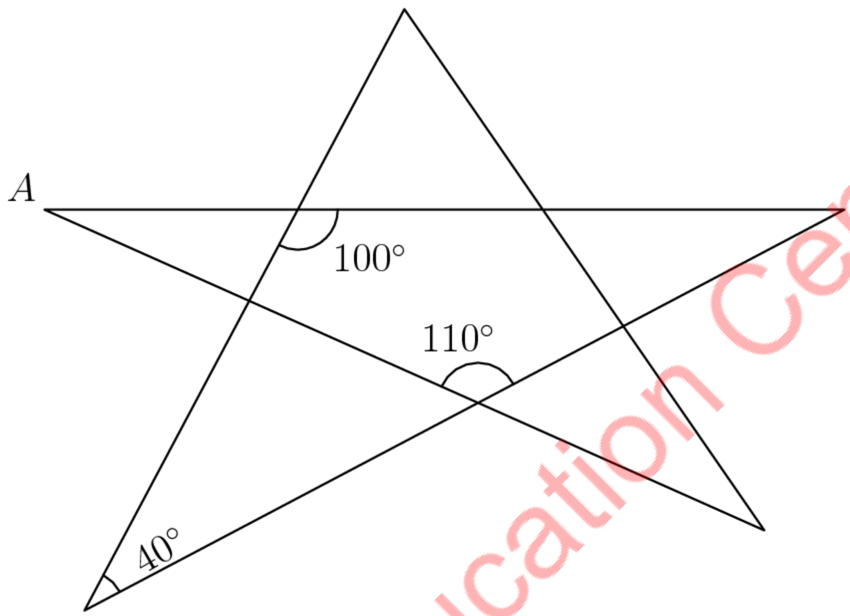


(A) 30 (B) 45 (C) 60 (D) 75 (E) 90

Problem Set V

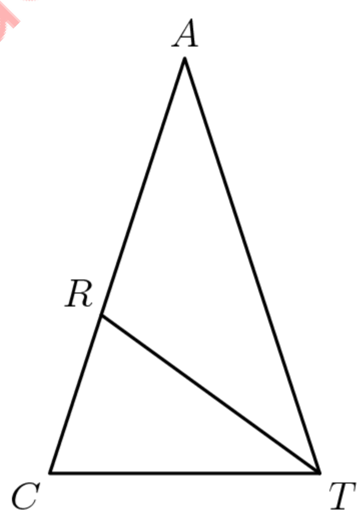
Problem 1: 1999 AMC 8 #21

The degree measure of angle A is



(A) 20 (B) 30 (C) 35 (D) 40 (E) 45

Problem 2: 2000 AMC 8 #13

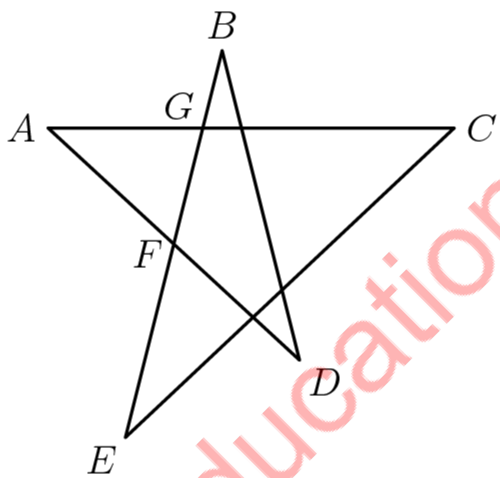


In triangle CAT, we have $\angle ACT = \angle ATC$ and $\angle CAT = 36^\circ$. If $\overline{TR}$ bisects $\angle ATC$, then $\angle CRT =$

(A) 36° (B) 54° (C) 72° (D) 90° (E) 108°

Problem 3: 2000 AMC 8 #24

If $\angle A = 20^\circ$ and $\angle AFG = \angle AGF$, then $\angle B + \angle D =$



(A) 48° (B) 60° (C) 72° (D) 80° (E) 90°

Problem 4: 2001 AMC 8 #13

Of the 36 students in Richelle's class, 12 prefer chocolate pie, 8 prefer apple, and 6 prefer blueberry. Half of the remaining students prefer cherry pie and half prefer lemon. For Richelle's pie graph showing this data, how many degrees should she use for cherry pie?

(A) 10 (B) 20 (C) 30 (D) 50 (E) 72

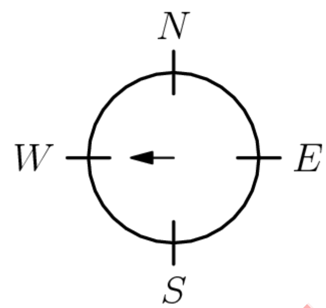
Problem 5: 2003 AMC 8 #20

What is the measure of the acute angle formed by the hands of the clock at 4:20 PM?

(A) 0 **(B)** 5 **(C)** 8 **(D)** 10 **(E)** 12

Problem 6: 2006 AMC 8 #4

Initially, a spinner points west. Chenille moves it clockwise $2\frac{1}{4}$ revolutions and then counterclockwise $3\frac{3}{4}$ revolutions. In what direction does the spinner point after the two moves?



$\textbf{(A)}\ \text{north} \qquad \textbf{(B)}\ \text{east} \qquad \textbf{(C)}\ \text{south} \qquad \textbf{(D)}\ \text{west} \qquad \textbf{(E)}\ \text{northwest}$

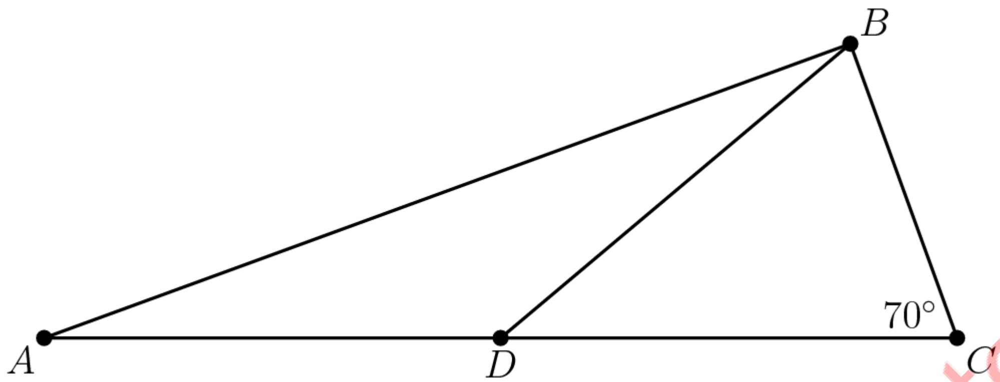
Problem 7: 2009 AMC 8 #19

Two angles of an isosceles triangle measure 70° and x°. What is the sum of the three possible values of x?

$\textbf{(A)}\ 95 \qquad \textbf{(B)}\ 125 \qquad \textbf{(C)}\ 140 \qquad \textbf{(D)}\ 165 \qquad \textbf{(E)}\ 180$

Problem 8: 2014 AMC 8 #9

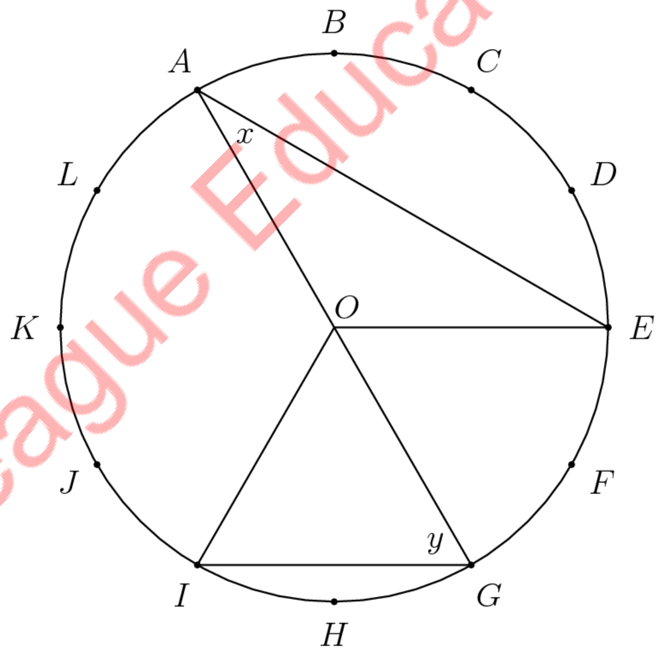
In $\triangle ABC$, D is a point on side $\overline{AC}$ such that $BD = DC$ and $\angle BCD$ measures 70°. What is the degree measure of $\angle ADB$?



$\textbf{(A)}\ 100 \qquad \textbf{(B)}\ 120 \qquad \textbf{(C)}\ 135 \qquad \textbf{(D)}\ 140 \qquad \textbf{(E)}\ 150$

Problem 9: 2014 AMC 8 #15

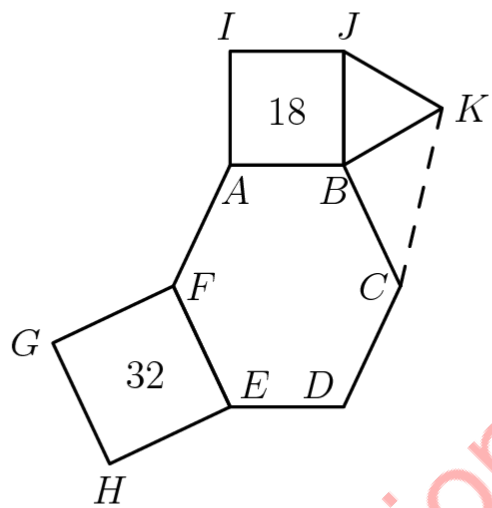
The circumference of the circle with center O is divided into 12 equal arcs, marked the letters A through L as seen below. What is the number of degrees in the sum of the angles x and y?



$\textbf{(A)}\ 75 \qquad \textbf{(B)}\ 80 \qquad \textbf{(C)}\ 90 \qquad \textbf{(D)}\ 120 \qquad \textbf{(E)}\ 150$

Problem 10: 2015 AMC 8 #21

In the given figure hexagon $ABCDEF$ is equiangular, $ABJI$ and $FEHG$ are squares with areas 18 and 32 respectively, $\triangle JBK$ is equilateral and $FE = BC$. What is the area of $\triangle KBC$?



$\textbf{(A)}\ 6\sqrt{2} \quad \textbf{(B)}\ 9 \quad \textbf{(C)}\ 12 \quad \textbf{(D)}\ 9\sqrt{2} \quad \textbf{(E)}\ 32$

.

Problem 11: 2016 AMC 8 #23

Two congruent circles centered at points A and B each pass through the other circle's center. The line containing both A and B is extended to intersect the circles at points C and D. The circles intersect at two points, one of which is E. What is the degree measure of $\angle CED$?

$\textbf{(A)}\ 90 \qquad \textbf{(B)}\ 105 \qquad \textbf{(C)}\ 120 \qquad \textbf{(D)}\ 135 \qquad \textbf{(E)}\ 150$

Problem 12: 2017 AMC 8 #6

If the degree measures of the angles of a triangle are in the ratio $3 : 3 : 4$, what is the degree measure of the largest angle of the triangle?

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 36 \qquad \textbf{(C)}\ 60 \qquad \textbf{(D)}\ 72 \qquad \textbf{(E)}\ 90$

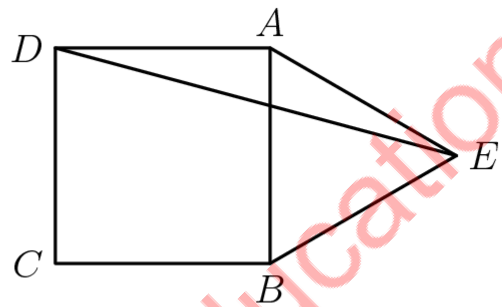
Bonus -- Problem Set VI

Problem 1: 1956 AHSME #10

A circle of radius 10 inches has its center at the vertex C of an equilateral $\triangle ABC$ and passes through the other two vertices. The side AC extended through C intersects the circle at D. The number of degrees of $\angle ADB$ is:

$\textbf{(A)}\ 15^\circ \qquad \textbf{(B)}\ 30^\circ \qquad \textbf{(C)}\ 60^\circ \qquad \textbf{(D)}\ 90^\circ \qquad \textbf{(E)}\ 120^\circ$

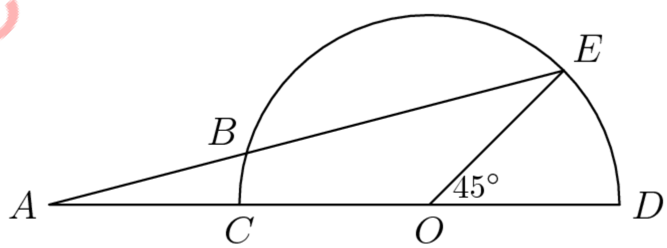
Problem 2: 1979 AHSME #3



In the adjoining figure, $ABCD$ is a square, ABE is an equilateral triangle and point E is outside square $ABCD$. What is the measure of $\measuredangle AED$ in degrees?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 12.5 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 25$

Problem 3: 1979 AHSME #12

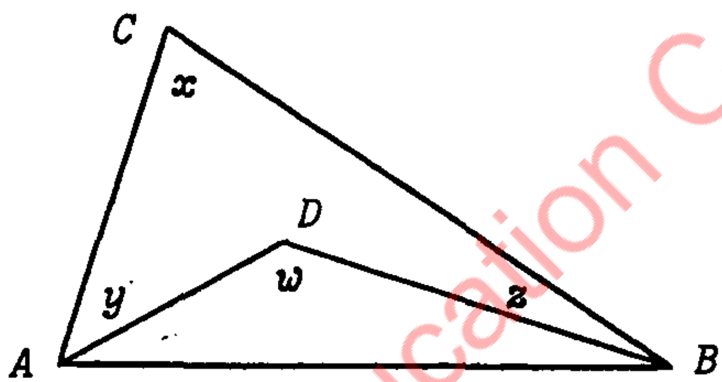


In the adjoining figure, CD is the diameter of a semi-circle with center O. Point A lies on the extension of DC past C; point E lies on the semi-circle, and B is the point of intersection (distinct from E) of line segment AE with the semi-circle. If length AB equals length OD, and the measure of $\measuredangle EOD$ is 45°, then the measure of $\measuredangle BAO$ is

(A) 10° **(B)** 15° **(C)** 20° **(D)** 25° **(E)** 30°

Problem 4: 1987 AHSME #6

In the $\triangle ABC$ shown, D is some interior point, and x, y, z, w are the measures of angles in degrees. Solve for x in terms of y, z and w.



(A) $w-y-z$ **(B)** $w-2y-2z$ **(C)** $180-w-y-z$
(D) $2w-y-z$ **(E)** $180-w+y+z$

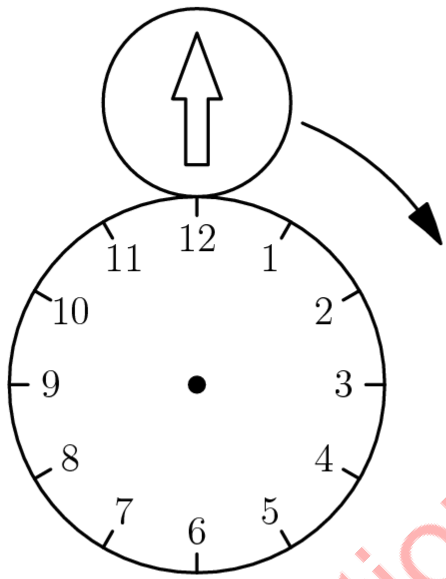
Problem 5: 2007 AMC 12A #6

Triangles ABC and ADC are isosceles with $AB = BC$ and $AD = DC$. Point D is inside triangle ABC, angle ABC measures 40 degrees, and angle ADC measures 140 degrees. What is the degree measure of angle BAD?

(A) 20 (B) 30 (C) 40 (D) 50 (E) 60

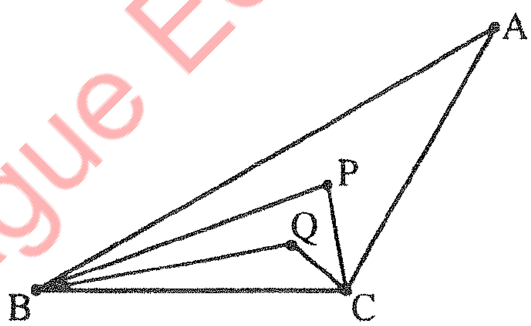
Problem 6: 2015 AMC 10A #14

The diagram below shows the circular face of a clock with radius 20 cm and a circular disk with radius 10cm externally tangent to the clock face at 12 o' clock. The disk has an arrow painted on it, initially pointing in the upward vertical direction. Let the disk roll clockwise around the clock face. At what point on the clock face will the disk be tangent when the arrow is next pointing in the upward vertical direction?



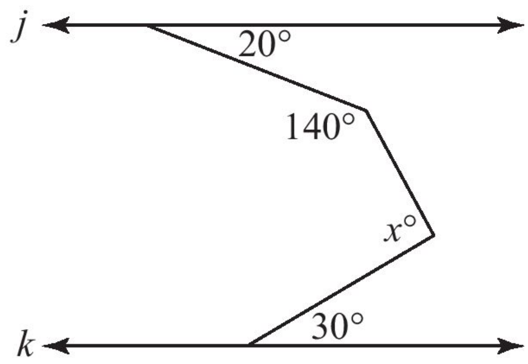
(A) 2 o' clock **(B)** 3 o' clock **(C)** 4 o' clock **(D)** 6 o' clock **(E)** 8 o' clock

Problem 7: 1994 MathCounts National Sprint #5



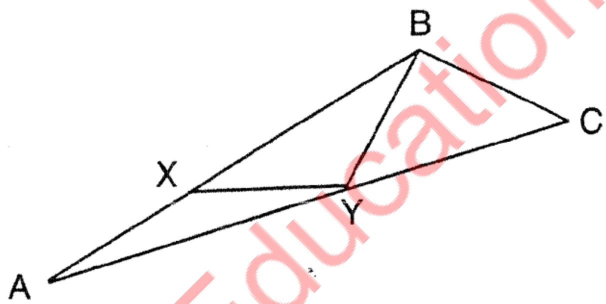
The trisectors of 2 angles of a scalene triangle ABC meet at points P and Q as shown. The third angle of the triangle, A, is 30 degrees. Find the measure in degrees of angle BPC.

Problem 8: 2014 MathCounts National Sprint #11



If $j \parallel k$, what is the value of x, in the figure shown?

Problem 9: 1993 MathCounts National Target #7



In triangle ABC, $AX = XY = YB = BC$ and the measure of angle ABC is 120 degrees. What is the degree measure of angle BAC?

Problem 10: 2004 MathCounts State Sprint #26

If all angles are measured in degrees, the ratio of three times the measure of $\angle A$ to four times the measure of the complement of $\angle A$ to half the measure of the supplement of $\angle A$ is $3 : 14 : 4$. What is the number of degrees in the measure of the complement of $\angle A$?

Problem 11: 2021 MAΘ National Convention Theta Triangles #5

Suppose the degree measures of the interior angles of a triangle form an arithmetic sequence. What is the middle term of this sequence?

(A) 30° (B) 90° (C) 45° (D) 60° (E) None of the Above

Problem 12: 2021 MAΘ National Convention Theta Triangles #8

The measures of the exterior angles of a triangle are in the ratio $4:5:6$. What is the ratio of the measures of their corresponding interior angles?

(A) $11:10:9$ (B) $7:5:3$ (C) $6:5:4$

(D) $5:4:3$ (E) None of the Above