2023 AMC 10A Problems

Problem 1

Cities A and B are 45 miles apart. Alice and Barbara start biking from A and B at speeds of 18 mph and 12 mph, respectively. How far away from city A will they be when they meet?

$\textbf{(A) }20\qquad\textbf{(B) }24\qquad\textbf{(C) }25\qquad\textbf{(D) }26\qquad\textbf{(E) }27$

Problem 2

The weight of $\frac{1}{3}$ of a large pizza together with $3\frac{1}{2}$ cups of orange slices is the same as the weight of $\frac{3}{4}$ of a large pizza together with $\frac{1}{2}$ cup of orange slices. A cup of orange slices weighs $\frac{1}{4}$ of a pound. What is the weight, in pounds, of a large pizza?

(A) $1\frac{4}{5}$ (B) 2 (C) $2\frac{2}{5}$ (D) 3 (E) $3\frac{3}{5}$

Problem 3

How many positive perfect squares less than 2023 are divisible by 5?

$\textbf{(A) }8\qquad\textbf{(B) }9\qquad\textbf{(C) }10\qquad\textbf{(D) }11\qquad\textbf{(E) }12$

Problem 4

A quadrilateral has all integer sides lengths, a perimeter of 26, and one side of length 4. What is the greatest possible length of one side of this quadrilateral?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

Problem 5

How many digits are in the base-ten representation of $8^5 \cdot 5^{10} \cdot 15^5$?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

Problem 6

An integer is assigned to each vertex of a cube. The value of an edge is defined to be the sum of the values of the two vertices it touches, and the value of a face is defined to be the sum of the values of the four edges surrounding it. The value of the cube is defined as the sum of the values of its six faces. Suppose the sum of the integers assigned to the vertices is 21. What is the value of the cube?

(A) 42 **(B)** 63 **(C)** 84 **(D)** 126 **(E)** 252

Problem 7

Janet rolls a standard 6-sided die 4 times and keeps a running total of the numbers she rolls. What is the probability that at some point, her running total will equal 3?

$\textbf{(A) } \frac{2}{9} \qquad \textbf{(B) } \frac{49}{216} \qquad \textbf{(C) } \frac{25}{108} \qquad \textbf{(D) } \frac{17}{72} \qquad \textbf{(E) } \frac{13}{54}$

Problem 8

Barb the baker has developed a new temperature scale for her bakery called the Breadus scale, which is a linear function of the Fahrenheit scale. Bread rises at 110 degrees Fahrenheit, which is 0 degrees on the Breadus scale. Bread is baked at 350 degrees Fahrenheit, which is 100 degrees on the Breadus scale. Bread is done when its internal temperature is 200 degrees Fahrenheit. What is this in degrees on the Breadus scale?

$\textbf{(A) } 33 \qquad \textbf{(B) } 34.5 \qquad \textbf{(C) } 36 \qquad \textbf{(D) } 37.5 \qquad \textbf{(E) } 39$

Problem 9

A digital display shows the current date as an 8-digit integer, consisting of a 4-digit year, followed by a 2-digit month, followed by a 2-digit date within the month. For how many dates in 2023 will each digit appear an even number of times in the digital display for that date?

$\textbf{(A) } 5 \qquad \textbf{(B) } 6 \qquad \textbf{(C) } 7 \qquad \textbf{(D) } 8 \qquad \textbf{(E) } 9$

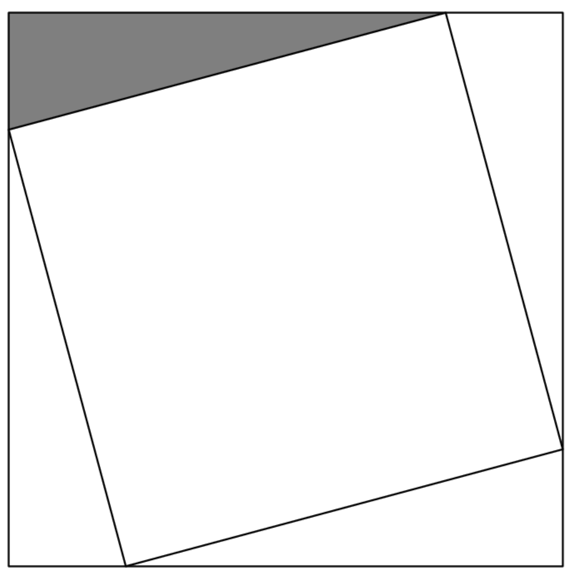
Problem 10

If Mareen scores an 11 on her next test, her mean score will go up by 1. If she gets three 11's in a row, her mean score will increase by 2. What is her current mean test score?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

Problem 11

A square with area 3 has a square with area 2 inscribed in it. This creates 4 smaller congruent right triangles. What is the ratio of the smaller leg to the larger leg in the shaded right triangle?



(A) $\frac{1}{5}$ **(B)** $\frac{1}{4}$ **(C)** $2-\sqrt{3}$ **(D)** $\sqrt{3}-\sqrt{2}$ **(E)** $\sqrt{2}-1$

Problem 12

How many three-digit positive integers N satisfy the following properties?

- The number N is divisible by 7.
- The number formed by reversing the digits of N is divisible by 5.

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

Problem 13

Abdul and Chiang are standing 48 feet apart in a field. Bharat is standing in the same field as far from Abdul as possible so that the angle formed by his lines of sight to Abdul and Chiang measures 60°. What is the square of the distance (in feet) between Abdul and Bharat?

(A) 1728 **(B)** 2601 **(C)** 3072 **(D)** 4608 **(E)** 6912

Problem 14

A number is chosen at random from among the first 100 positive integers, and a positive integer divisor of that number is then chosen at random. What is the probability that the chosen divisor is divisible by 11?

(A) $\frac{4}{100}$ **(B)** $\frac{9}{200}$ **(C)** $\frac{1}{20}$ **(D)** $\frac{11}{200}$ **(E)** $\frac{3}{50}$

Problem 15

An even number of circles are nested, starting with a radius of 1 and increasing by 1 each time, all sharing a common point. The region between every other circle is shaded, starting with the region inside the circle of radius 2 but outside the circle of radius 1. An example showing 8 circles is displayed below. What is the least number of circles needed to make the total shaded area at least 2023π?

(A) 46 (B) 48 (C) 56 (D) 60 (E) 64

Problem 16

In a tennis tournament, each person plays every other person once. In this tournament, there are twice as many right-handed players than left-handed players, but left-handed players won 40% more games than right-handed players. How many total games were played?

(A) 15 **(B)** 36 **(C)** 45 **(D)** 48 **(E)** 66

Problem 17

Let $ABCD$ be a rectangle with $AB = 30$ and $BC = 28$. points P and q lie on $\overline{BC}$ and $\overline{CD}$ respectively so that all sides of $\Delta ABP, \Delta PCQ,$ and ΔQDA have integer lengths. what is the perimeter of ΔAPQ?

(A) 84 (B) 86 (C) 88 (D) 90 (E) 92

Problem 18

A rhombic dodecahedron is a solid with 12 congruent rhombus faces. At every vertex, 3 or 4 edges meet, depending on the vertex. How many vertices have exactly 3 edges meet?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

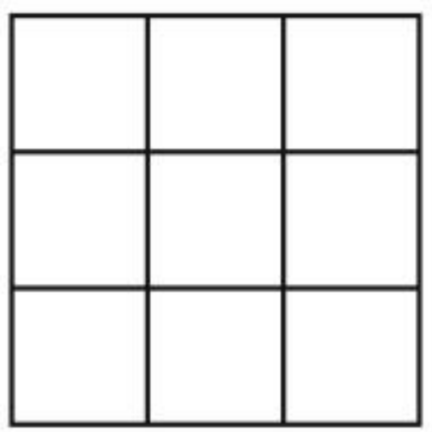
Problem 19

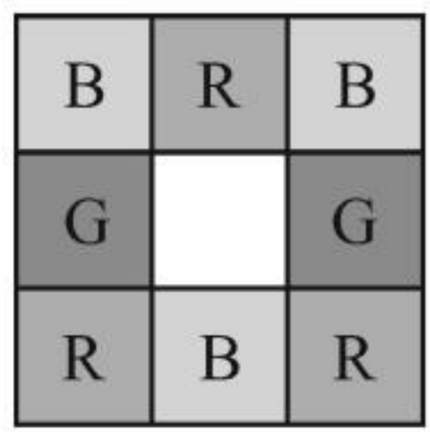
The line segment formed by $A(1,2)$ and $B(3,3)$ is rotated to the line segment formed by $A'(3,1)$ and $B'(4,3)$ about the point $P(r,s)$. What is $|r-s|$?

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{3}{4}$ D) $\frac{2}{3}$ E) 1

Problem 20

Each square in a 3×3 grid of squares is colored red, white, blue, or green so that every 2×2 square contains one square of each color. One such coloring is shown on the right below. How many different colorings are possible?





$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 48 \qquad \textbf{(C)}\ 60 \qquad \textbf{(D)}\ 72 \qquad \textbf{(E)}\ 96$

Problem 21

Consider the polynomial $P(x)$ such that:

- 1 is a root of $P(x) - 1$,
- 2 is a root of $P(x-2)$,
- 3 is a root of $P(3x)$, and
- 4 is a root of $4P(x)$.

All the roots of $P(x)$ except one are integers. If the one non-integer root can be written in the form $\frac{m}{n}$, where m and n are relatively prime, what is $m + n$?

(A) 41 **(B)** 43 **(C)** 45 **(D)** 47 **(E)** 49

Problem 22

Circle C_1 and C_2 have radius 1, and the distance between their centers is $\frac{1}{2}$. Circle C_3 is the largest circle internally tangent to both C_1 and C_2. Circle C_4 is internally tangent to both C_1 and C_2 and is externally tangent to C_3. What is the radius of C_4?

(A) $\frac{1}{14}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{3}{28}$ **(E)** $\frac{1}{9}$

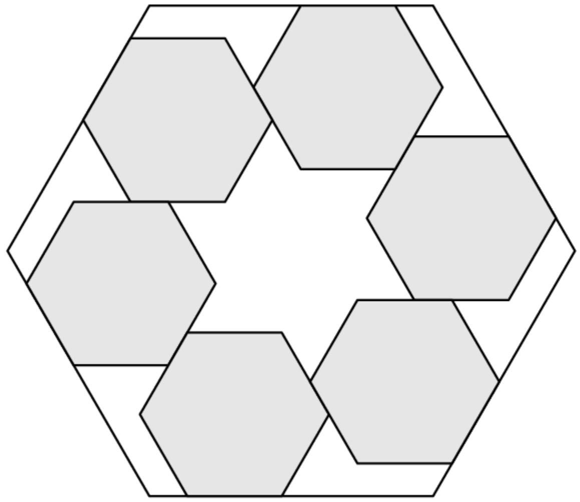
Problem 23

Positive integer divisors a and b of n are called complementary if $ab = n$. Given that N has a pair of complementary divisors that differ by 20 and a pair of complementary divisors that differ by 23, find the sum of the digits of N.

(A) 11 **(B)** 13 **(C)** 15 **(D)** 17 **(E)** 19

Problem 24

Six regular hexagonal blocks of side length 1 unit are arranged inside a regular hexagonal frame. Each block lies along an inside edge of the frame and is aligned with two other blocks, as shown in the figure below. The distance from any corner of the frame to the nearest vertex of a block is $\frac{3}{7}$ unit. What is the area of the region inside the frame not occupied by the blocks?



(A) $\frac{13\sqrt{3}}{3}$ **(B)** $\frac{216\sqrt{3}}{49}$ **(C)** $\frac{9\sqrt{3}}{2}$ **(D)** $\frac{14\sqrt{3}}{3}$ **(E)** $\frac{243\sqrt{3}}{49}$

Problem 25

If A and B are vertices of a polyhedron, define the distance $d(A,B)$ to be the minimum number of edges of the polyhedron one must traverse in order to connect A and B. For example, if $\overline{AB}$ is an edge of the polyhedron, then $d(A,B)=1$, but if $\overline{AC}$ and $\overline{CB}$ are edges and $\overline{AB}$ is not an edge, then $d(A,B)=2$. Let $Q, R,$ and S be randomly chosen distinct vertices of a regular icosahedron (regular polyhedron made up of 20 equilateral triangles). What is the probability that $d(Q,R)>d(R,S)$?

(A) $\frac{7}{22}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{5}{12}$ **(E)** $\frac{1}{2}$

Answer Key

1. E
2. A
3. A
4. D
5. E
6. D
7. B
8. D
9. E
10. D
11. C
12. B
13. C
14. B
15. E
16. B
17. A
18. D
19. E
20. D
21. D
22. D
23. C
24. C
25. A